

**Best
Available
Copy**

The US Army's Center for Strategy and Force Evaluation

RESEARCH PAPER
CAA-RP-94-2

AD-A280 498



①

**PERSONNEL ATTRITION RATES IN
HISTORICAL LAND COMBAT OPERATIONS:
A NOTE ON THE PROBABILITY OF
READMISSIONS AND MULTIPLE WOUNDS**

DTIC
ELECTE
S F D
JUN 23 1994

APRIL 1994



DTIC STAFF INSPECTED 2

**PREPARED BY
TACTICAL ANALYSIS DIVISION**

**US ARMY CONCEPTS ANALYSIS AGENCY
8120 WOODMONT AVENUE
BETHESDA, MARYLAND 20814-2797**

94-19211



528

This document has been approved
for public release and sale; its
distribution is unlimited.

94 6 22 126

CAA

DISCLAIMER

The findings of this report are not to be construed as an official Department of the Army position, policy, or decision unless so designated by other official documentation. Comments or suggestions should be addressed to:

**Director
US Army Concepts Analysis Agency
ATTN: CSCA-TCT
8120 Woodmont Avenue
Bethesda, MD 20814-2797**

REPORT DOCUMENTATION PAGE			Form Approved OPM No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources gathering and maintaining the data needed, and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Information and Regulatory Affairs, Office of Management and Budget, Washington, DC 20503.				
1. AGENCY USE ONLY (Leave Blank)		2. REPORT DATE April 1994		3. REPORT TYPE AND DATES COVERED Final, Feb 94 - Apr 94
4. TITLE AND SUBTITLE Personnel Attrition Rates in Historical Land Combat Operations: A Note on the Probability of Readmissions and Multiple Wounds			5. FUNDING NUMBERS (Work unit accession number from DD1498)	
6. AUTHOR(S) Robert L. Helmbold				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) US Army Concepts Analysis Agency 8120 Woodmont Avenue Bethesda, MD 20814-2797			8. PERFORMING ORGANIZATION REPORT NUMBER CAA-RP-94-2	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited			12b. DISTRIBUTION CODE A	
13. ABSTRACT (Maximum 200 words) This paper develops a model of the probability of multiple incidence. The model is based on an application of the Maximum Entropy Principle. The predictions of this model are tested against data on the number of aircraft with multiple personnel casualties, the number of casualties with multiple anatomical regions wounded, the number of personnel with multiple wounds, and the number of personnel with multiple WIA admissions. It is shown that in several instances, the model gives reasonably accurate fits to the data. This model will be useful to those engaged in weapons systems analysis and development, wargaming and simulation, studies of military medical and personnel support systems, and the assessment of personal protective devices.				
14. SUBJECT TERMS Personnel, combat, attrition, battle, casualties, losses			15. NUMBER OF PAGES 59	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

**PERSONNEL ATTRITION RATES IN HISTORICAL
LAND COMBAT OPERATIONS:
A NOTE ON THE PROBABILITY OF
READMISSIONS AND MULTIPLE WOUNDS**

April 1994

Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

Prepared by
Dr. Robert L. Helmbold
TACTICAL ANALYSIS DIVISION
US Army Concepts Analysis Agency
8120 Woodmont Avenue
Bethesda, Maryland 20814-2797

PREFACE

The Personnel Attrition Rates (PAR) Study as a whole is limited to studying personnel strengths and battle casualties in historical land combat operations. Other types of attrition (nonbattle losses, losses to equipment, casualties to other services, and so forth) are outside PAR's scope, as are personnel losses in models, simulations, wargames, field experiments, or training exercises (like those of the National Training Center).

Phase 1, or PAR-P1, was devoted to assembling the available data and past studies on personnel strengths and attrition rates in land combat operations, preparing a comprehensive bibliography of it, and planning the approach to subsequent phases. Its specific objectives were to:

- Collect as many as possible of the available tabulated data and data-based studies of attrition rates in historical land combat operations,
- Prepare a comprehensive bibliography of such data and studies, and
- Outline an approach to accomplishing the subsequent phases of the PAR Study as a whole.

The bibliography of works collected during Phase 1 was published as *Personnel Attrition Rates in Land Combat Operations: An Annotated Bibliography*, US Army Concepts Analysis Agency Research Paper, CAA-RP-93-2, June 1993 (AD-A268 787). The collection of data and data-based studies consists of the files of pertinent documents maintained at the US Army Concepts Analysis Agency.

Phases 2 and 3 of the PAR Study will convert some of the most important data to electronic form in order to facilitate its analysis, and will perform selected analyses of the attrition data to derive information useful in US Army wargames, studies, and analyses. As of this writing, the following publications have been published during Phase 2:

- "Personnel Attrition Rates in Historical Land Combat Operations: Susceptibility and Vulnerability of Major Anatomical Regions," CAA Research Paper CAA-RP-93-3, August 1993, AD-A270 766.
- "Personnel Attrition Rates in Historical Land Combat Operations: A Catalog of Attrition and Casualty Data Bases on Diskettes Usable With Personal Computers," CAA Research Paper CAA-RP-93-4, September 1993.

This paper, written as part of Phase 2, furnishes an additional analysis. It develops a model, based on the Maximum Entropy Principle, of the probability of multiple incidence. The predictions of this model are tested against data on the number of aircraft with multiple personnel casualties, the number of casualties with multiple anatomical regions wounded, the number of personnel with multiple wounds, and the number of personnel with multiple WIA admissions. It is shown that in several instances, the model gives moderately accurate fits to the data. This model will be useful to those engaged in weapons systems analysis and development, wargaming and simulation, studies of military medical and personnel support systems, and the assessment of personal protective devices.



REPLY TO
ATTENTION OF
CSCA-TCT

DEPARTMENT OF THE ARMY

US ARMY CONCEPTS ANALYSIS AGENCY
8120 WOODMONT AVENUE
BETHESDA, MARYLAND 20814-2797



23 MAY 1984

**MEMORANDUM FOR DEPUTY UNDER SECRETARY OF THE ARMY (OR),
HEADQUARTERS, DEPARTMENT OF THE ARMY,
WASH DC 20310-0544**

**SUBJECT: Personnel Attrition Rates in Historical Land Combat Operations:
Note on the Probability of Readmissions and Multiple Wounds**

1. The U.S. Army Concepts Analysis Agency (CAA) is pleased to publish this Research Paper by Dr. Robert L. Helmbold. It develops a model of the probability of multiple incidence based on an application of the Maximum Entropy Principle. The predictions of this model are tested against data on the number of aircraft with multiple personnel casualties, the number of casualties with multiple anatomical regions wounded, the number of personnel with multiple wounds, and the number of personnel with multiple WIA admissions. It is shown that in several instances, the model gives moderately accurate fits to the data. This model will be useful to those engaged in weapons systems analysis and development, wargaming and simulation, studies of military medical and personnel support systems, and the assessment of personal protective devices. Its publication will make this work available to others for further use in their work.
2. Questions or inquiries should be directed to the U.S. Army Concepts Analysis Agency (ATTN: CSCA-TCT), 8120 Woodmont Avenue, Bethesda, MD 20814-2797, (301) 295-1611 or DSN 295-1611.

E. B. VANDIVER III
Director



**PERSONNEL ATTRITION RATES IN
HISTORICAL LAND COMBAT
OPERATIONS: A NOTE ON THE
PROBABILITY OF READMISSIONS AND
MULTIPLE WOUNDS**

**SUMMARY
CAA-RP-94-2**

THE REASON FOR PREPARING THIS PAPER is that a model of the probability of multiple incidence is needed to analyze data on the number of aircraft with multiple personnel casualties, the number of casualties with multiple anatomical regions wounded, the number of personnel with multiple wounds, and the number of personnel with multiple wounded in action (WIA) admissions.

THE SPONSOR is the Director, US Army Concepts Analysis Agency (CAA).

THE OBJECTIVE is to provide the Army with a model of the probability of multiple incidence suitable for application to cases of the number of aircraft with multiple personnel casualties, the number of casualties with multiple anatomical regions wounded, the number of personnel with multiple wounds, and the number of personnel with multiple WIA admissions.

THE SCOPE OF THE STUDY is limited to the probability of multiple incidence in the situations mentioned above.

THE MAIN ASSUMPTION of this paper is that the bulk of the pertinent works have been collected and are on file at CAA.

THE BASIC APPROACH is to compare the distributions of multiple incidence obtained from an application of the Maximum Entropy Principle to the observed distributions.

THE PRINCIPAL FINDINGS of this work are that the distributions of multiple incidence obtained from the Maximum Entropy Principle agree reasonably well with historical data when such data is available for comparison.

THE STUDY EFFORT was directed by Dr. Robert L. Helmbold, Tactical Analysis Division.

COMMENTS AND QUESTIONS may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-TCT, 8120 Woodmont Avenue, Bethesda, Maryland 20814-2797.

(THIS PAGE INTENTIONALLY LEFT BLANK)

CONTENTS

PREFACE

CHAPTER

Page

1	EXECUTIVE SUMMARY.....	1-1
	Background.....	1-1
	Objective	1-1
	Scope	1-1
	Assumptions	1-2
	Approach	1-2
	Findings and Observations	1-2
2	DISCUSSION OF A PARTICULAR EXAMPLE.....	2-1
	Introduction	2-1
	An Initial Example	2-1
	A Second Example	2-2
3	RESULTS AND DISCUSSION	3-1
	Introduction	3-1
	Distribution of Aircraft Taking Casualties.....	3-1
	Distribution of Anatomical Regions Affected.....	3-5
	Distribution of Number of Wounds	3-13
	Distribution of the Number of Admissions.....	3-16
4	FINDINGS AND OBSERVATIONS	4-1

APPENDIX

A	Bibliography	A-1
B	Remarks on the Maximum Entropy Principle	B-1
C	Distribution	C-1

GLOSSARY	Glossary-1
-----------------------	-------------------

FIGURES

FIGURE		Page
2-1	Fitted and Observed Distribution of Regions Wounded	2-3
2-2	Estimated Distribution of Wounds for the Turkish Brigade Example	2-4
3-1	Observed and Fitted Values for Beyer's Table 177	3-2
3-2	Observed and Fitted Values for Beyer's Table 178	3-3
3-3	Observed and Fitted Values for Beyer's Table 179	3-4
3-4	Observed and Fitted Values for Beyer's Table 182 (Pooled)	3-6
3-5	Observed and Fitted Values for Beyer's Table 182 (B-17s)	3-7
3-6	Observed and Fitted Values for Beyer's Table 182 (B-24s)	3-8
3-7	Observed and Fitted Values for Beyer's Table 188 (WIA)	3-9
3-8	Observed and Fitted Values for Beyer's Table 188 (KIA)	3-10
3-9	Observed and Fitted Values for Beyer's Table 188 (WIA+KIA)	3-11
3-10	Observed and Fitted Values for Beyer's Table 221	3-12
3-11	Observed and Fitted Values for McBride's Data	3-14
3-12	Observed and Fitted Values for Uhorchak's Data	3-15
3-13	Observed and Fitted Values for AEF WIA Admissions	3-17
3-14	Observed and Fitted Values for Krivosheyev's Officer Data	3-18
3-15	Observed and Fitted Values for Krivosheyev's Sergeant Data	3-19
3-16	Observed and Fitted Values for Krivosheyev's Soldier Data	3-20
3-17	Observed and Fitted Values for Krivosheyev's Data on all Ranks	3-21

CHAPTER 1

EXECUTIVE SUMMARY

1-1. BACKGROUND. In April 1992, the US Army Concepts Analysis Agency (CAA) started a three-phased study of Personnel Attrition Rates (PAR). The present document covers only the portion of this work having to do with selected aspects of the probability of readmissions and multiple wounds.

1-2. OBJECTIVE. The main reason for performing this study was to put on record the work done on an application of the Maximum Entropy Principle to estimating the distribution of multiple events. Specifically, the multiple events under study are the number of aircraft with multiple personnel casualties, the number of casualties with multiple anatomical regions wounded, the number of personnel with multiple wounds, and the number of personnel with multiple WIA admissions. The issue addressed is, "What do the data tell us about the validity of estimates of the distribution of multiple events obtained from an application of the Maximum Entropy Principle?" The Maximum Entropy Principle states that the most likely probability distribution within a given class is the one with maximum entropy (see Appendix B for a more detailed discussion of the Maximum Entropy Principle and its application to the kinds of data treated in this paper).

1-3. SCOPE. The PAR Study as a whole is limited to studying the personnel strengths and battle casualties of land combat forces. Other types of attrition (nonbattle losses, losses to equipment, casualties to other services, and so forth) are outside PAR's scope. PAR is concerned only with historical data on actual combat operations: it will *not* deal with personnel losses in models, simulations, wargames, field experiments, or training exercises (such as those of the National Training Center). PAR focuses mainly on either original or translated works in English, although some important work in other languages may be included. Studies of personnel attrition are also included, provided they contain cogent analyses of a publicly available, nonproprietary body of tabulated data on attrition in actual combat operations. Since trends in attrition over long periods of time are of interest, data on ancient as well as recent battles are solicited. However, as no contract support is anticipated and in-house resources are limited, no systematic effort is made to extract data from the archives or primary source materials, and no original historical research is envisioned. Thus, PAR relies almost exclusively on secondary works that contain data in readily usable tabulated form. All works received prior to the cutoff date of 31 May 1993 are included in the final report on Phase 1 (see CAA-1993, in the Bibliography at Appendix A).

The scope of the present paper is generally limited to wounds inflicted on personnel by projectile impact, where "projectiles" include bullets, shell fragments, flechettes, shrapnel, grapeshot, and similar items. It is (at least in principle) possible to locate the anatomical site of the injuries caused by such projectiles. Bodily injury inflicted by weapons or weapon effects that are difficult to localize are included to the extent that they are included in the existing historical data on combat casualties. Some examples of the types of weapons or weapon effects excluded are chemical and biological weapons such as war gases and other toxic substances, nuclear weapon effects (blast, ionizing radiation, and thermal effects), and directed energy weapons. Injuries to personnel in armored vehicles are not specifically included, primarily because sufficient data to perform a proper analysis of that case was not in hand.

1-4. ASSUMPTIONS. The main assumption of this paper is that the bulk of the pertinent works have been collected and are on file at CAA.

1-5. APPROACH. The basic approach is to test estimates for the distributions of multiple events obtained from the Maximum Entropy Principle by comparing them to historical data.

1-6. FINDINGS AND OBSERVATIONS. It is feasible to use published data to test the estimated distributions of multiple events obtained from the Maximum Entropy Principle. Moreover, such comparisons show that the estimated distributions obtained from the Maximum Entropy Principle often agree moderately well with historical data when such data is available for comparison. The fits occasionally would be rejected by the rote application of statistical hypothesis tests, but on the other hand these data often do not satisfy all of the assumptions required for the applicability of formal statistical hypothesis tests. At the same time, visual inspection shows that the fit is usually about as good as might be expected, and is often adequate for practical use when moderately accurate estimates are better than having no estimates. Although the results are very supportive of the applicability of the Maximum Entropy Principle to these kinds of problems, the available historical data are meager. Additional relevant data would be most welcome.

CHAPTER 2

DISCUSSION OF A PARTICULAR EXAMPLE

2-1. INTRODUCTION. As stated in Chapter 1, the issue to be addressed is, "What do the data tell us about the validity of estimates of the distributions of multiple events obtained from an application of the Maximum Entropy Principle?" Here we will discuss a particular example, taken from [Beyer-1962, Table 227, p 601], to illustrate the general approach. Additional examples are given in Chapter 3.

2-2. AN INITIAL EXAMPLE. Beyer, *op cit.*, gives the following information:

- There are 164 killed in action (KIA) personnel involved.
- The wounds received have been divided into a total of five categories according to the number of major anatomical regions that received wounds.
- The total number of anatomical regions wounded is 294. This is the sum of the number of personnel wounded, weighted by the number of anatomical regions in which they were wounded. Thus, an individual wounded in exactly two anatomical regions contributes a count of two to the total number of anatomical regions wounded, and so forth.

Now it is evident from the above information that some of the personnel must have been wounded in more than one anatomical region. Suppose we ask how many of the personnel were wounded in exactly one, exactly two, *etc.*, anatomical regions. It is clear that the answer to this question is not uniquely determined by the information mentioned so far. Instead, some additional information or structure must be employed if a unique answer is to be obtained. If we appeal to the Maximum Entropy Principle (*see* Appendix B) and assume a maximum of five anatomical regions wounded per KIA, then there is a unique answer, which is as follows (rounded to the nearest integer in order to simplify the results for illustrative purposes):

- 87 of the KIA were wounded in exactly 1 anatomical region.
- 42 of the KIA were wounded in exactly 2 anatomical regions.
- 21 of the KIA were wounded in exactly 3 anatomical regions.
- 9 of the KIA were wounded in exactly 4 anatomical regions.
- 5 of the KIA were wounded in exactly 5 anatomical regions.

It can be seen that the Maximum Entropy solution agrees with the total number of KIA given by Beyer, *op cit*. It also agrees with the total of 294 anatomical regions wounded given by Beyer, *op cit*, except for a slight discrepancy due to the rounding.

The next obvious question is how well these values agree with reality. Often we have no way of knowing for sure how well the Maximum Entropy Principle gives results that agree with observation. In this particular case, however, Beyer, *op cit*, gives the following values:

- 82 of the KIA wounded in exactly 1 anatomical region.
- 49 wounded in exactly 2 regions.
- 22 wounded in exactly 3 regions.
- 7 wounded in exactly 4 regions.
- 4 wounded in exactly 5 regions.

Figure 2-1 compares these observed data to the values fitted by the Maximum Entropy Principle. It is clear that in this case, the distribution of regions wounded estimated by the Maximum Entropy Principle is a good approximation to reality.

The next obvious question is whether this case is unusual, or whether Maximum Entropy estimates reliably give fairly good approximations to reality. Chapter 3 presents several comparisons of Maximum Entropy estimates to the observed data. These comparisons show that the Maximum Entropy estimates usually are fairly close to the reported observations. The natural inference is that Maximum Entropy estimates will be fairly accurate even in cases where we lack the data for detailed comparison. An example of this situation is given in the next paragraph.

2-3. A SECOND EXAMPLE. Here we use data from [Beyer-1962, Table 281, p 715] which mentions 950 wounds in 286 Turkish WIA casualties who were interviewed at the Tokyo Army Hospital after evacuation from combat during the Korea War. (We note in passing that, due to their evacuation from the theater this group must have had more wounds per individual than the generality of wounded individuals in the Korean War.) The Maximum Entropy estimate for this group of Turkish WIA casualties, assuming no limit on the number of wounds per WIA, is shown on Figure 2-2. Although they seem to be reasonable, unfortunately Beyer, *op cit*, provides no detailed data for comparison.

Beyer Table 227, Page 601

Distribution of Regions by KIA

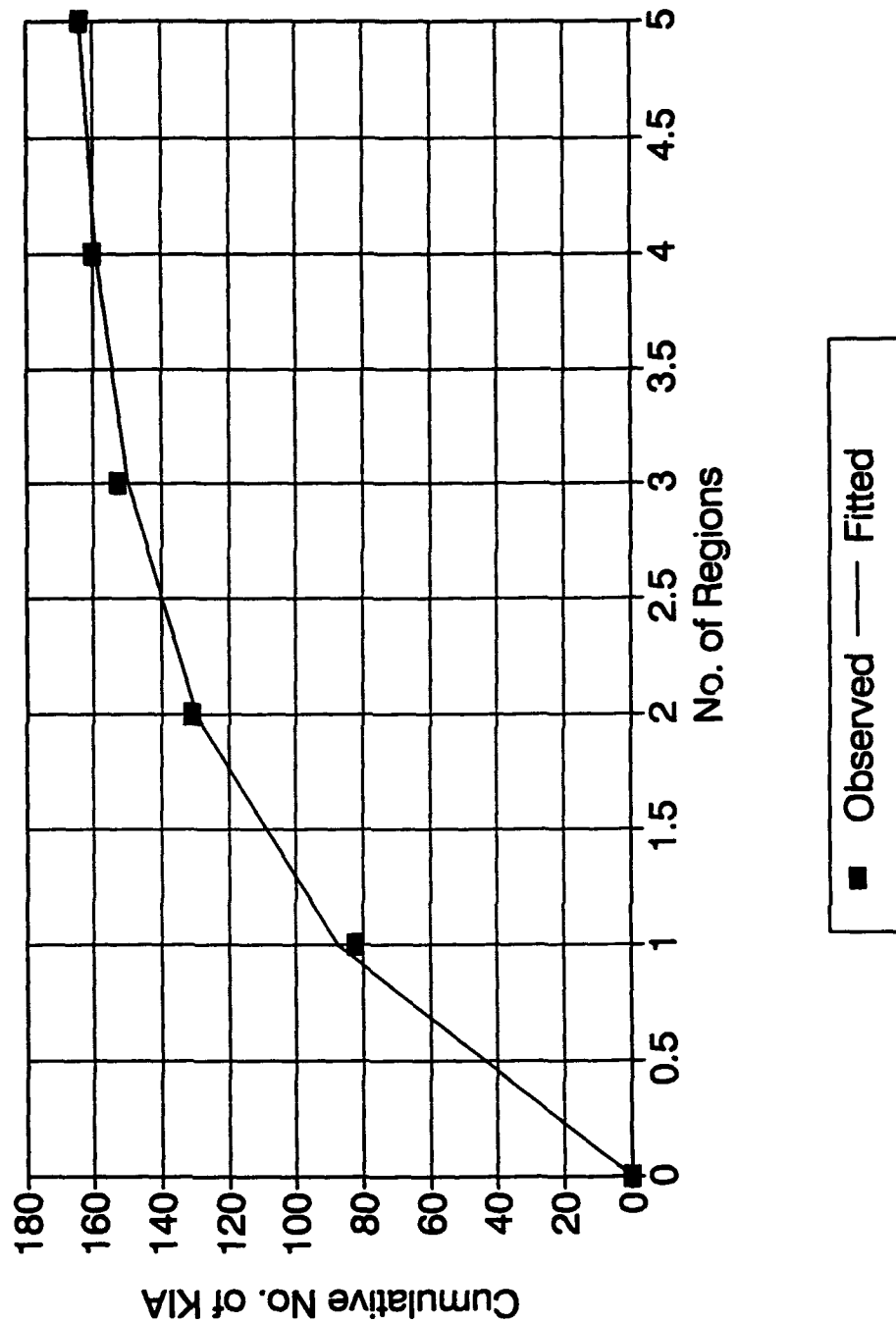


Figure 2-1. Fitted and Observed Distribution of Regions Wounded

Turkish Brigade Patients Distribution of Patients by Wounds

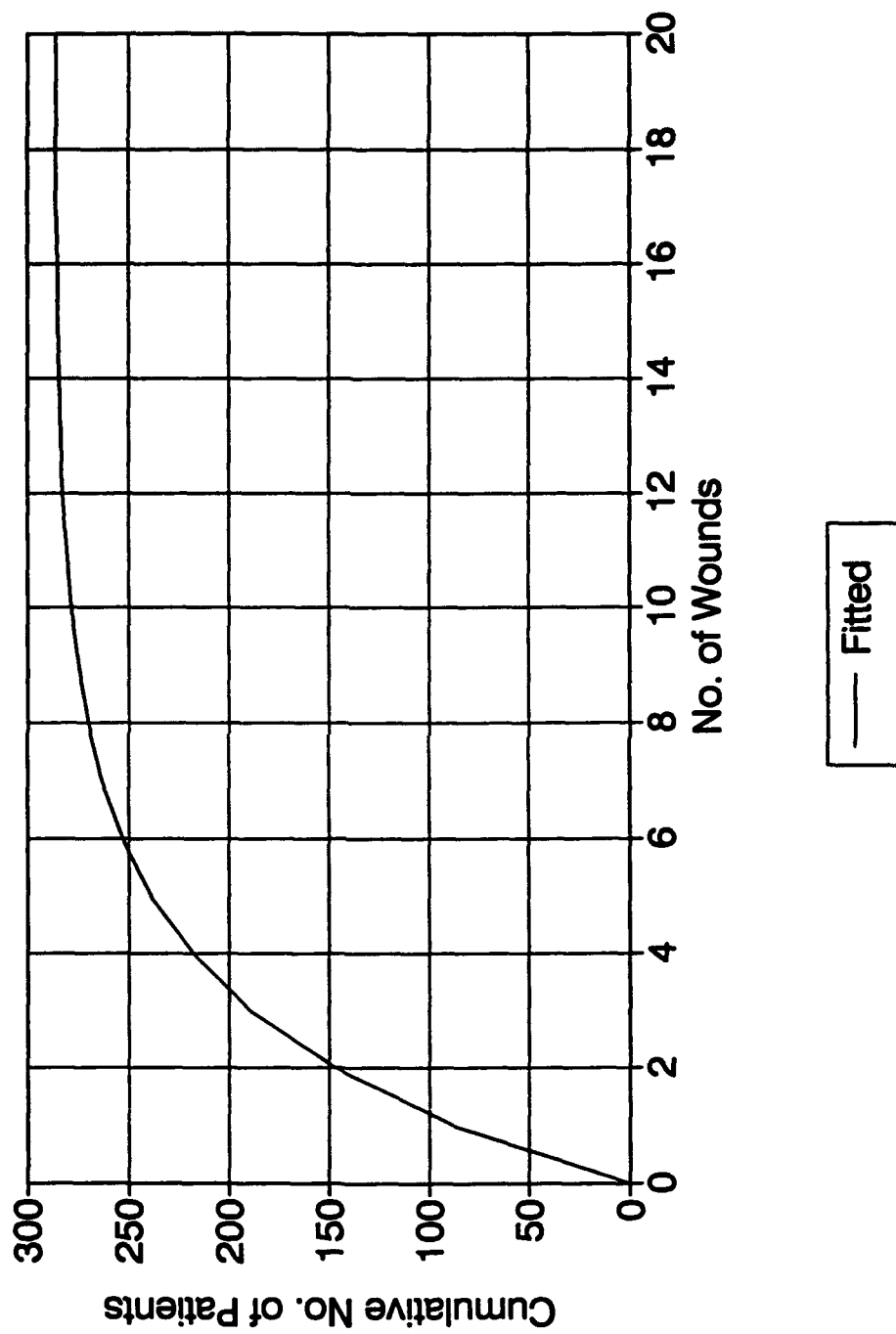


Figure 2-2. Estimated Distribution of Wounds for the Turkish Brigade Example

CHAPTER 3

RESULTS AND DISCUSSION

3-1. INTRODUCTION. This chapter illustrates the results of applying the Maximum Entropy Principle method described in Appendix B to some of the published data. For convenience, this chapter is divided into paragraphs corresponding to the following areas: distribution of aircraft by number of personnel casualties per aircraft, distribution of number of anatomical regions wounded by wounds per anatomical region, distribution of WIA by wounds per WIA, and distribution of wounded admissions by admissions per wounded individual.

3-2. DISTRIBUTION OF AIRCRAFT TAKING CASUALTIES. These examples compare the estimated number of aircraft taking n casualties to the observed number of aircraft taking n casualties. For these examples, the estimated number of aircraft taking n casualties is determined by the Maximum Entropy Principle using only the total number of aircraft, the average number of casualties per aircraft, and an assumed maximum number of casualties per aircraft.

a. Beyer [Beyer-1962, Table 177, p 553] gives a table showing the distribution of 541 casualties in 461 flak-damaged B-17 aircraft, where the number of casualties varies from 1 to 4. Figure 3-1 shows the Maximum Entropy estimate and reported number of aircraft taking n casualties for $n = 1(1)4$, assuming no more than four casualties per aircraft. The agreement appears to be satisfactory. Here and elsewhere in this paper, the notation $x = a(b)c$ means that x is to be varied from a to c in steps of b . Thus, $n = 1(1)4$ means that n is the take on the values 1, 2, 3, and 4.

b. Beyer [Beyer-1962, Table 178, p 554] gives a table showing the distribution of 193 casualties in 172 flak-damaged B-24 aircraft, where the number of casualties varies from 1 to 3. Figure 3-2 shows the Maximum Entropy estimate and reported number of aircraft taking n casualties for $n = 1(1)3$, assuming no more than three casualties per aircraft. The agreement appears to be satisfactory.

c. Beyer [Beyer-1962, Table 179, p 554] gives a table showing the distribution of 28 casualties in 19 B-17 aircraft damaged by missiles from fighter aircraft, where the number of casualties reported varies from 1 to 3. Figure 3-3 shows the Maximum Entropy estimate and reported number of aircraft taking n casualties for $n = 1(1)4$, assuming no more than 4 casualties per aircraft. The agreement seems satisfactory.

Beyer Table 177, Page 553 (B-17s) **Distribution of Aircraft by Casualties**

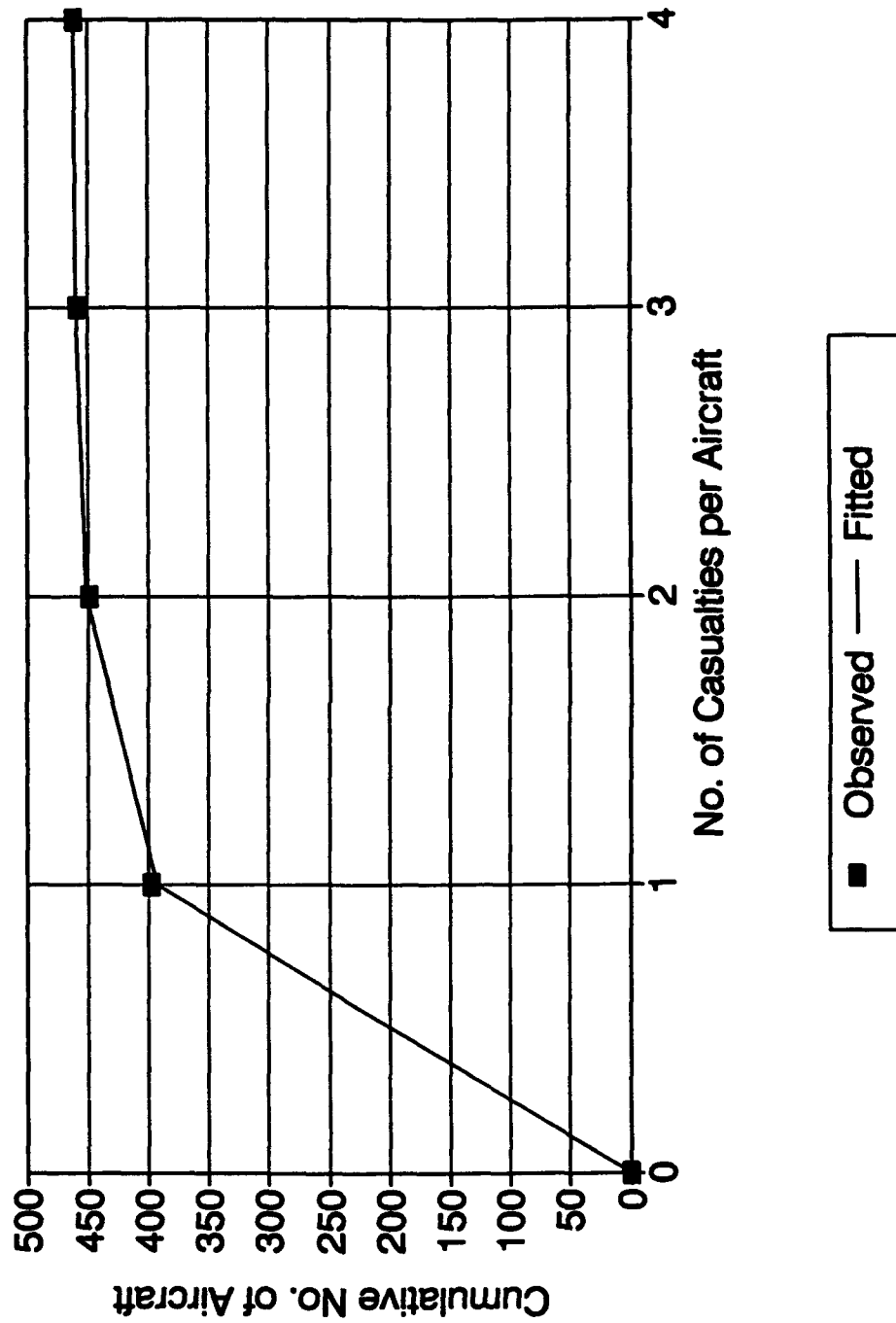


Figure 3-1. Observed and Fitted Values for Beyer's Table 177 (B-17s)

Beyer Table 178, Page 554 (B-24s)

Distribution of Aircraft by Casualties

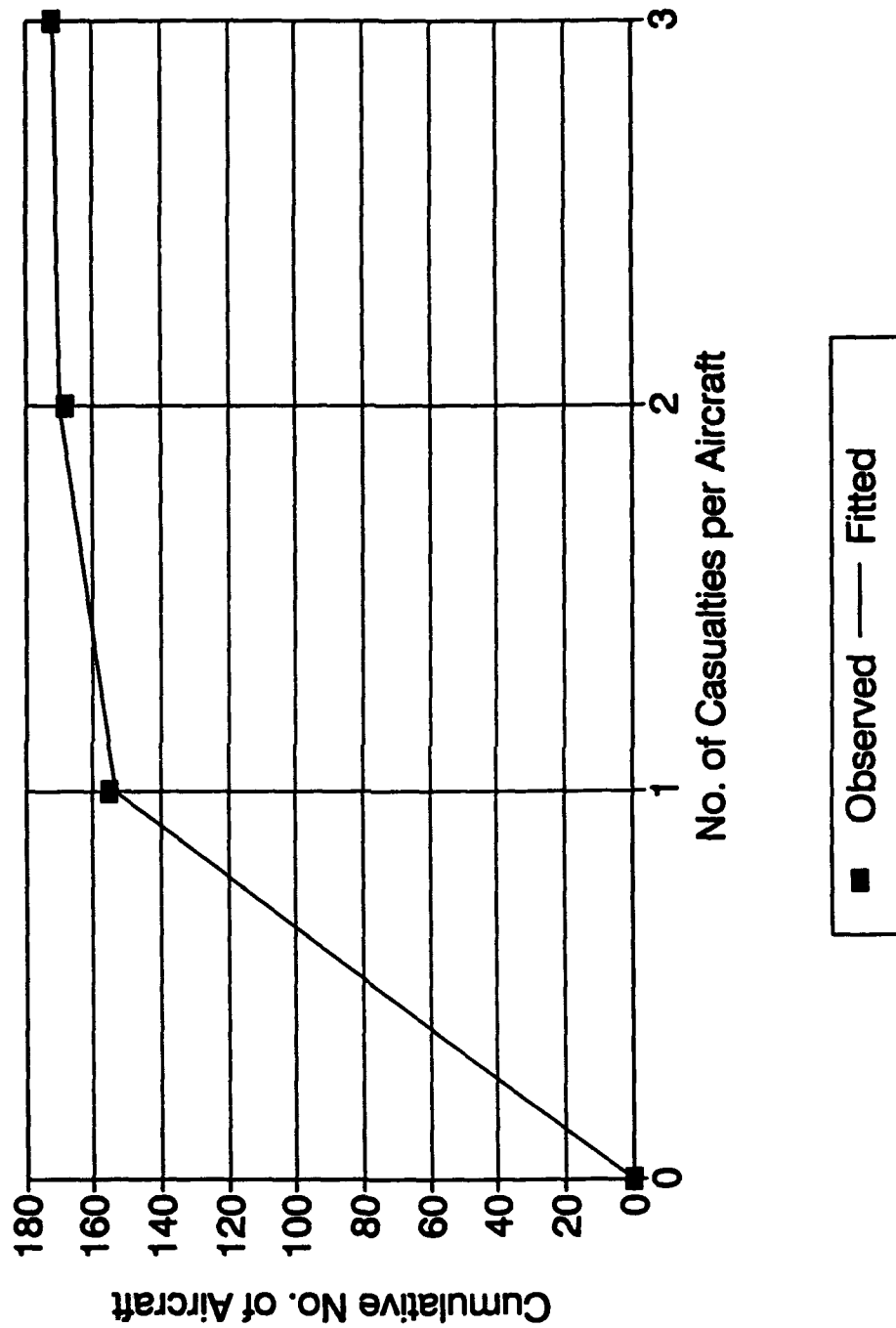


Figure 3-2. Observed and Fitted Values for Beyer's Table 178 (B-24s)

Beyer Table 179, Page 554 (B-17s) **Distribution of Aircraft by Casualties**

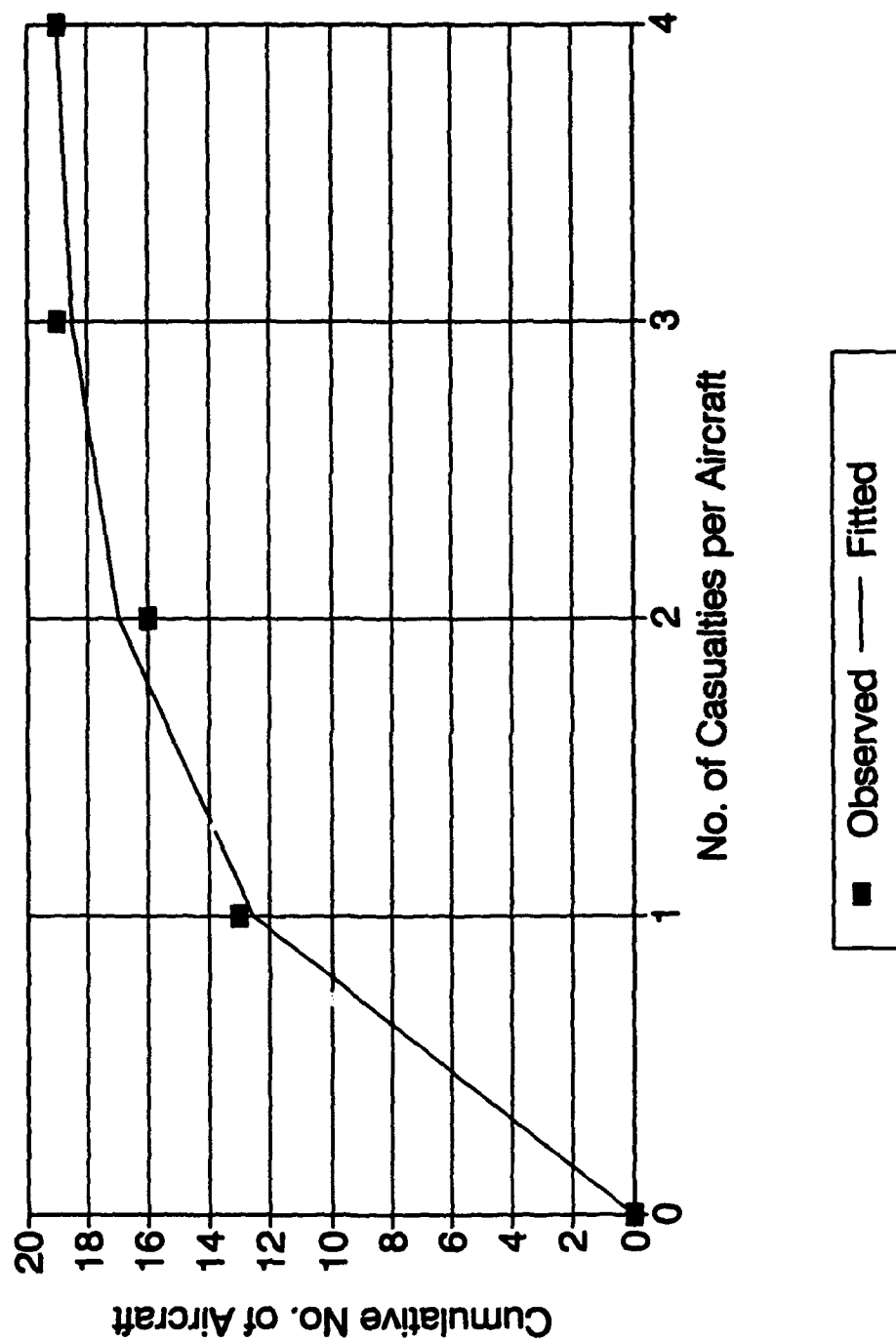


Figure 3-3. Observed and Fitted Values for Beyer's Table 179 (B-17s)

d. Beyer [Beyer-1962, Table 182, p 556] gives a table showing the distribution of 1,117 casualties, by category (WIA, KIA, and total casualties), in 944 heavy bombers (B-17s and B-24s). Applying the Maximum Entropy Principle to the total casualties and the pooled data for both types of heavy bomber and assuming no more than six casualties per aircraft produces the fit shown in Figure 3-4. The agreement seems satisfactory. Figure 3-5 shows the results for just the B-17s when we use total casualties and assume no more than five casualties per aircraft. These results also seem satisfactory. Figure 3-6 shows the results for just the B-24s when we use total casualties and assume no more than six casualties per aircraft. Again the agreement seems satisfactory. Good agreement was also obtained when casualty categories were separated into KIA and WIA.

3-3. DISTRIBUTION OF ANATOMICAL REGIONS AFFECTED. These examples compare the estimated number of casualties having n anatomical regions affected to the observed number of casualties having n anatomical regions affected. For these examples, the estimated number of casualties having n anatomical regions affected is determined by the Maximum Entropy Principle using only the total number of casualties, the average number of anatomical regions affected per casualty, and an assumed maximum number of anatomical regions affected.

a. Beyer [Beyer-1962, Table 188, p 565] gives the distribution of 1,117 air crew battle casualties by category (WIA, KIA, and total casualties = WIA + KIA) and by number of regions wounded. In these data, the number of regions wounded is limited to five. There are a total of 1,157 regions wounded for the 1,007 WIA cases, 174 regions for the 110 KIA cases, and 1,331 regions for the 1,117 total casualty cases. Figures 3-7 through 3-9 show the results of applying the Maximum Entropy Principle, assuming no more than five anatomical regions affected. The degree of agreement appears satisfactory.

b. Beyer [Beyer-1962, Table 221, p 597] gives the distribution of 50 air crew battle casualties due to missiles fired from fighter aircraft by category (WIA, KIA, and total) and number of anatomical regions wounded. The number of regions wounded is limited to four. There is a total of 83 regions wounded for the 50 casualties. Figure 3-10 shows the result of applying the Maximum Entropy Principle, assuming no more than four anatomical regions affected. The degree of agreement appears satisfactory.

c. Beyer [Beyer-1962, Table 227, p 601] gives the distribution of 164 KIA air crew battle casualties by number of anatomical regions wounded. In these data, the number of regions wounded is limited to five. There is a total of 294 regions wounded for the 164 casualties. Figure 2-1 of Chapter 2 shows the result of applying the Maximum Entropy Principle, assuming no more than five anatomical regions affected. The agreement seems satisfactory.

Beyer Table 182, Page 556 (Pooled)

Distribution of Aircraft by Casualties

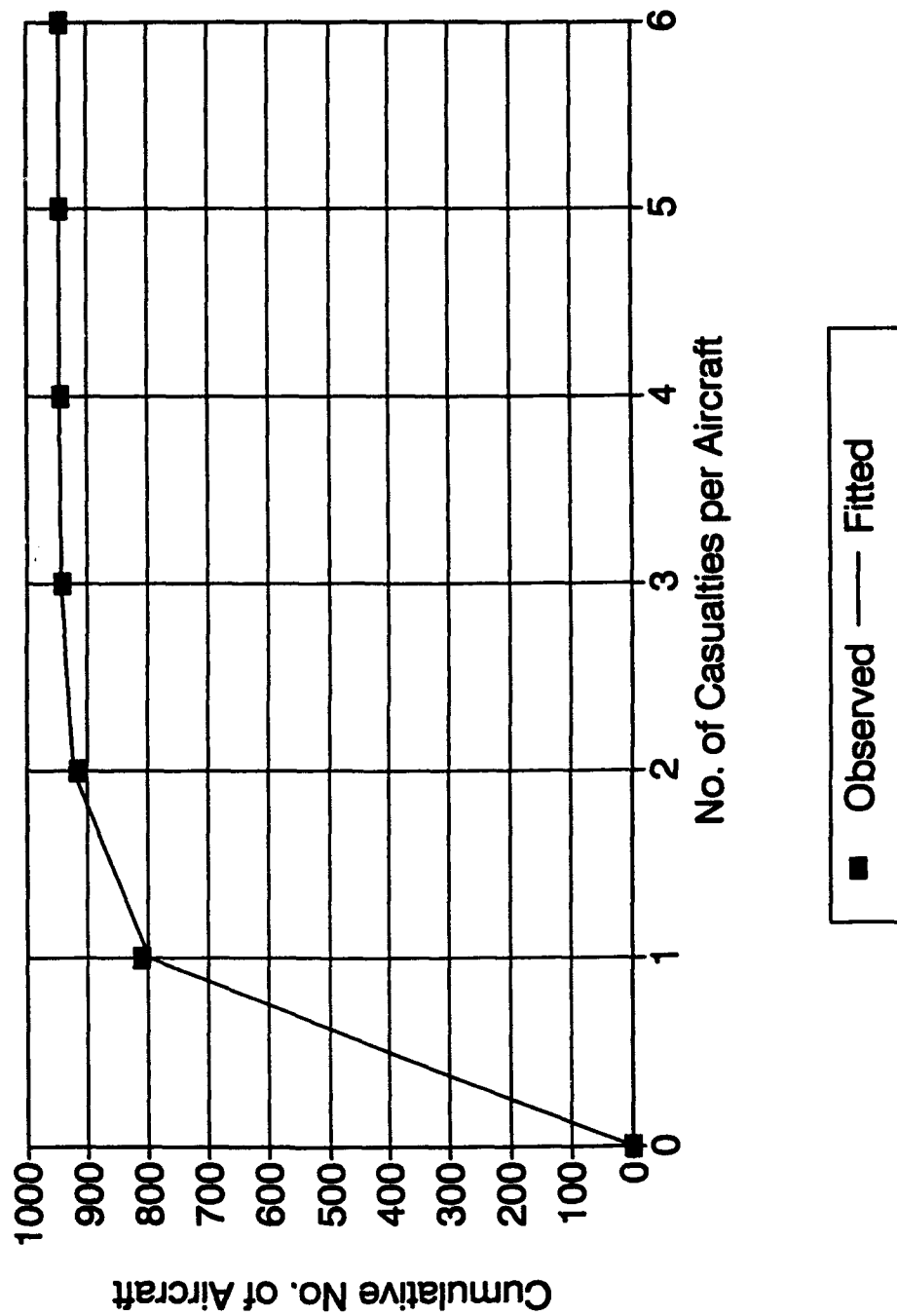


Figure 3-4. Observed and Fitted Values for Beyer's Table 182 (Pooled)

Beyer Table 182, Page 556 (B-17s)

Distribution of Aircraft by Casualties

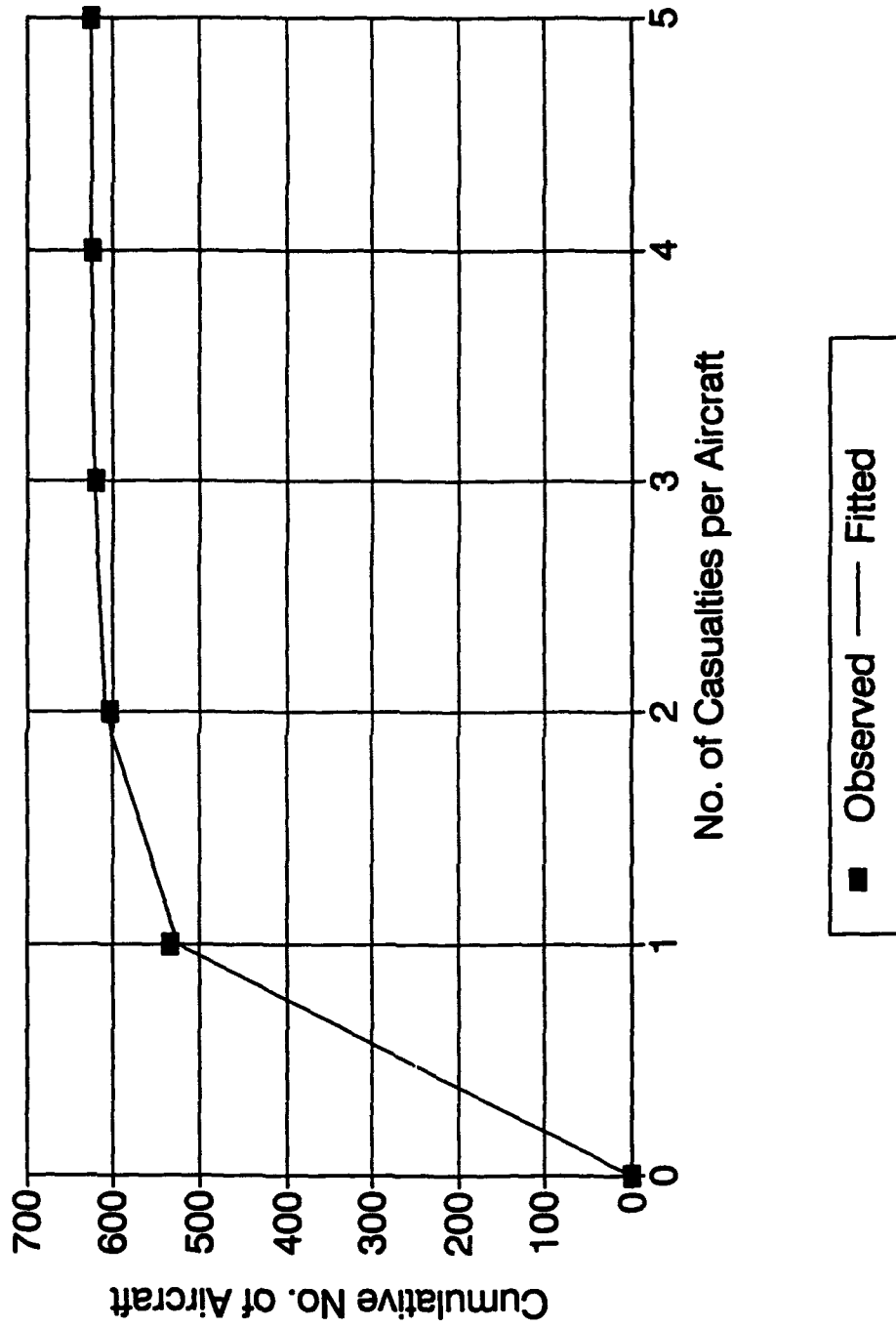


Figure 3-5. Observed and Fitted Values for Beyer's Table 182 (B-17s)

Beyer Table 182, Page 556 (B-24s)

Distribution of Aircraft by Casualties

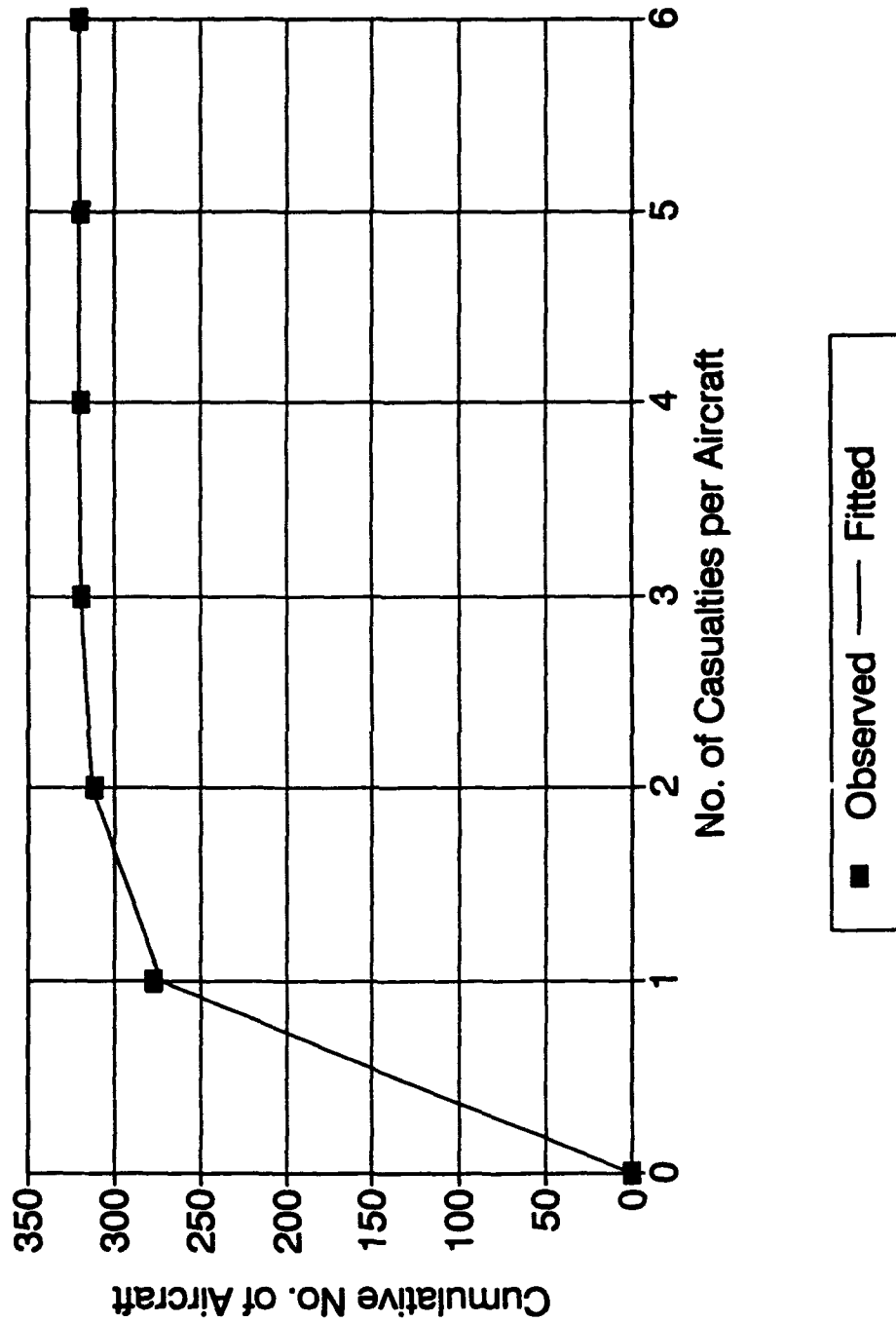


Figure 3-6. Observed and Fitted Values for Beyer's Table 182 (B-24s)

Beyer Table 188, Page 565 (WIA)

Distribution of WIA by Region

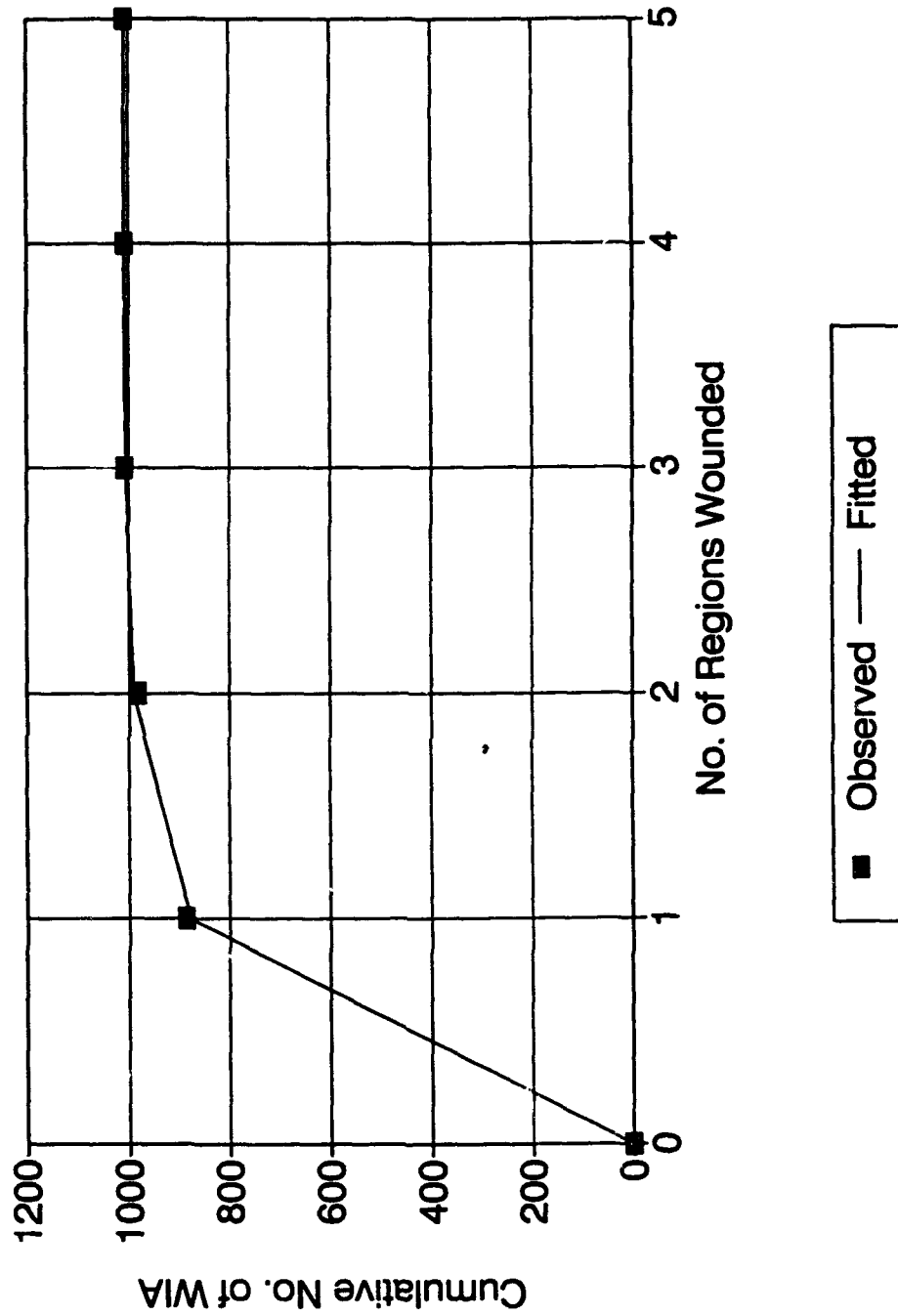


Figure 3-7. Observed and Fitted Values for Beyer's Table 188 (WIA)

Beyer Table 188, Page 565 (KIA)

Distribution of KIA by Region

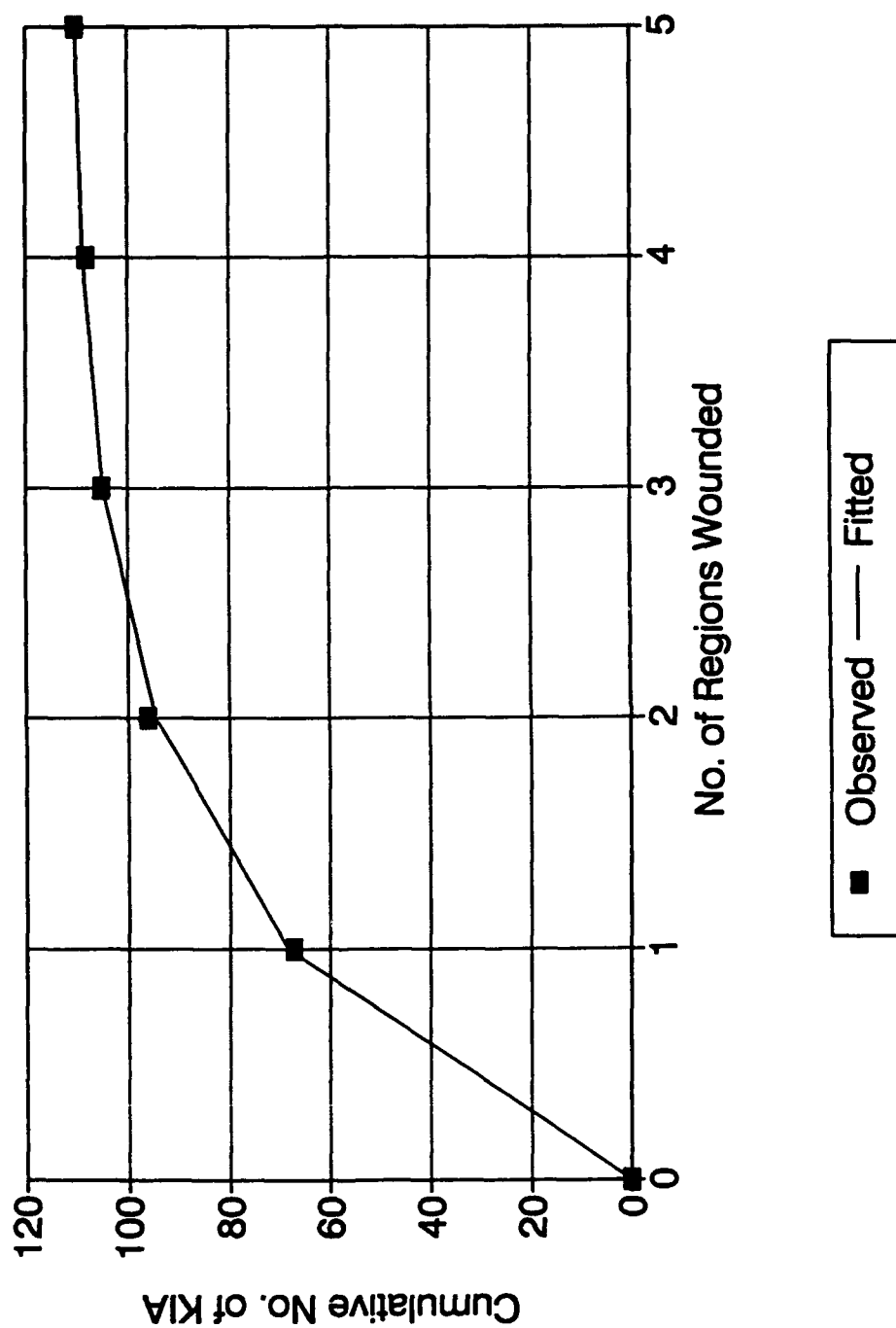


Figure 3-8. Observed and Fitted Values for Beyer's Table 188 (KIA).

Beyer Table 188, Page 565 (WIA+KIA)

Distribution of WIA+KIA by Region

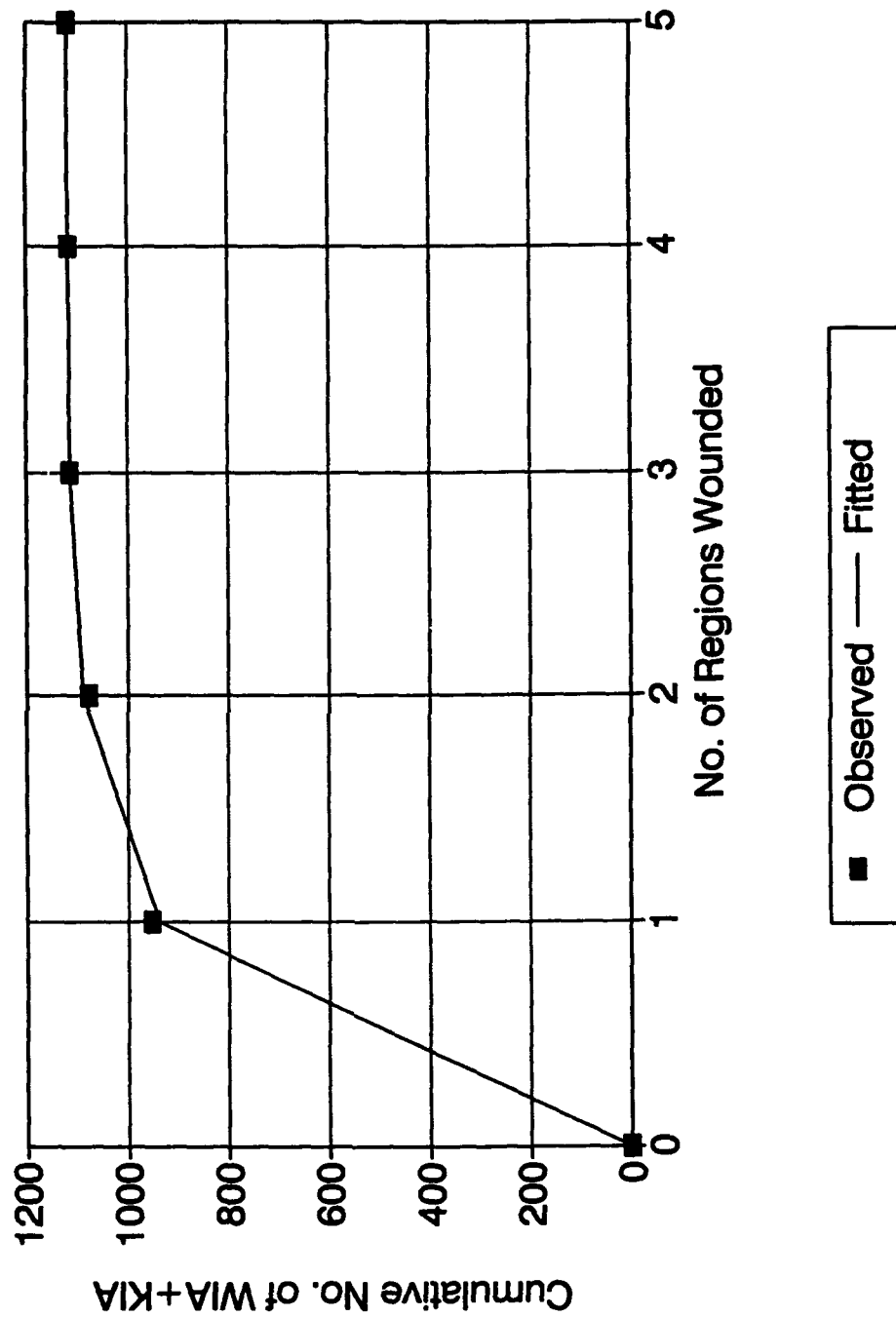


Figure 3-9. Observed and Fitted Values for Beyer's Table 188 (WIA+KIA)

Beyer Table 221, Page 597

Distribution of Casualties by Region

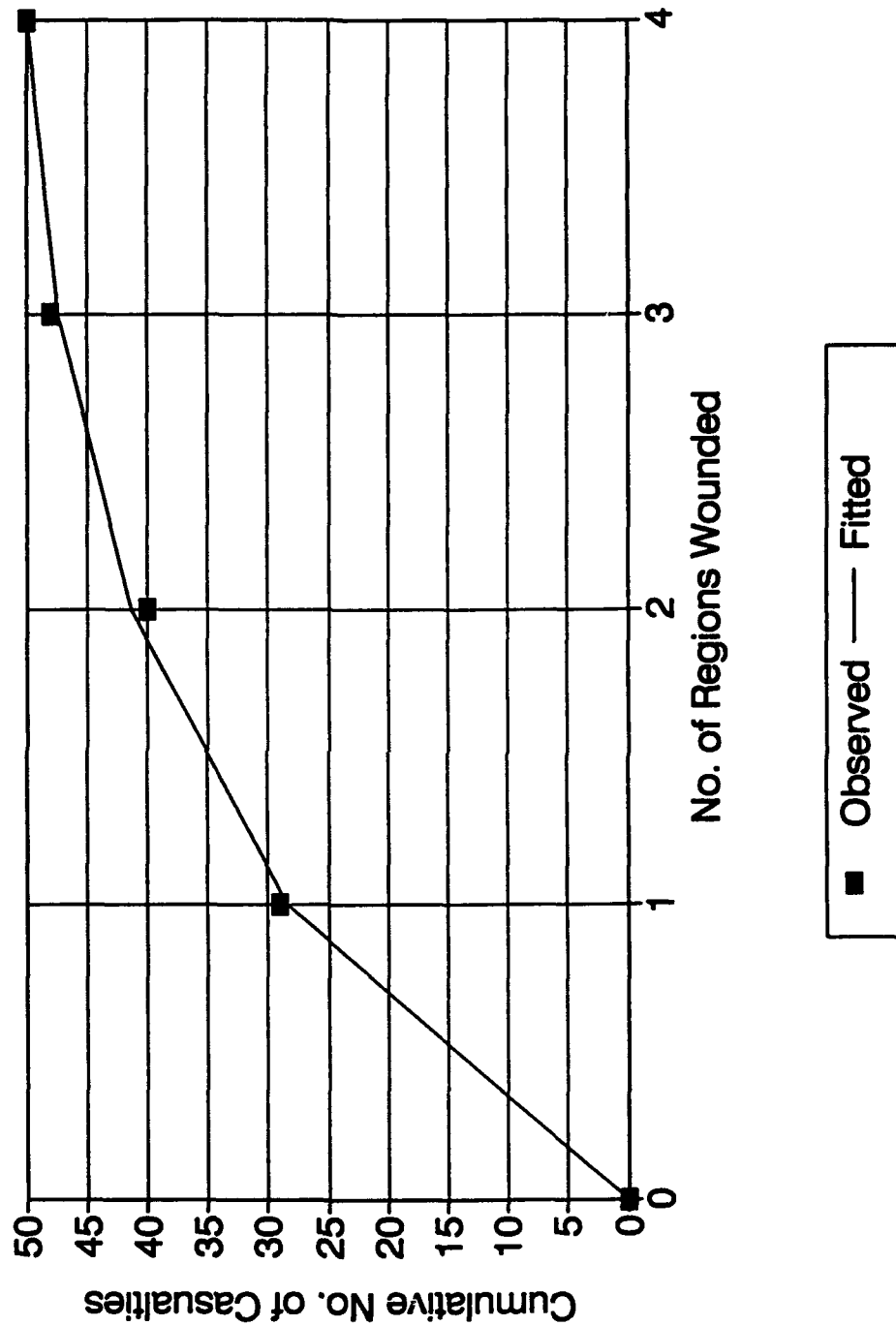


Figure 3-10. Observed and Fitted Values for Beyer's Table 221.

3-4. DISTRIBUTION OF NUMBER OF WOUNDS. These examples compare the estimated number of casualties having n wounds with the observed number of casualties having n wounds. For these examples, the estimated number of casualties having n wounds is determined by the Maximum Entropy Principle using only the total number of casualties, the average number of wounds per casualty, and an assumed maximum number of wounds to any particular casualty.

a. Beyer [Beyer-1962, Table 172, p 542] gives some information on the distribution of 133 wounds in 100 casualties. Applying the Maximum Entropy Principle, assuming no limit on the possible number of wounds per casualty, leads to an estimate of about 75 casualties with exactly 1 wound and 25 with multiple wounds. Beyer, *op cit*, records 70 casualties with exactly 1 wound and 30 with multiple wounds. The difference is not considered particularly significant, considering the relatively small size of the sample (100 casualties).

b. Beyer [Beyer-1962, p 706] gives some information on the distribution of 7,773 wounds in 4,600 WIA casualties. Assuming no limit on the number of wounds per casualty, the Maximum Entropy Principle leads to an estimate of 2,722 single wounds and 5,051 multiple wounds. Beyer, *op cit*, gives 2,621 single wounds, 4,846 multiple wounds, and 306 cases with an unknown number of wounds. Assuming that the 306 cases with an unknown number of wounds had single or multiple wounds in the same proportion as the 7,467 cases where the number of wounds was known leads to the estimates of $2,621 + 306 \times 2,621/7,467 = 2,728.4$ single wounds and $7,773 - 2,728.4 = 5,044.6$ multiple wounds. These estimates are in excellent agreement with those derived from the Maximum Entropy Principle.

c. McBride [McBride-1991, Table 7 and Figure 4] gives information on the distribution of injuries to casualties incurred during Operation JUST CAUSE (Panama, December 1989). There were 378 physical injuries suffered by 236 patients (the caption of McBride's Figure 4, which indicates 253 casualties, is in error because the numbers shown elsewhere in that figure sum to 236 casualties). Figure 3-11 shows the results of applying the Maximum Entropy Principle, assuming the number of injuries to a given patient are unlimited. The agreement is acceptable.

d. Uhorchak [Uhorchak-1992, Figure 4] gives information on the distribution of 472 injuries to 204 patients interviewed at US Army General Hospitals in Germany in the aftermath of Operation DESERT STORM (Kuwait Theater, January 1991). Figure 3-12 shows the results of applying the Maximum Entropy Principle, assuming that the number of injuries to a given patient are unlimited. The agreement is acceptable.

McBride Table 7 & Figure 4

Distribution of Patients by Wounds

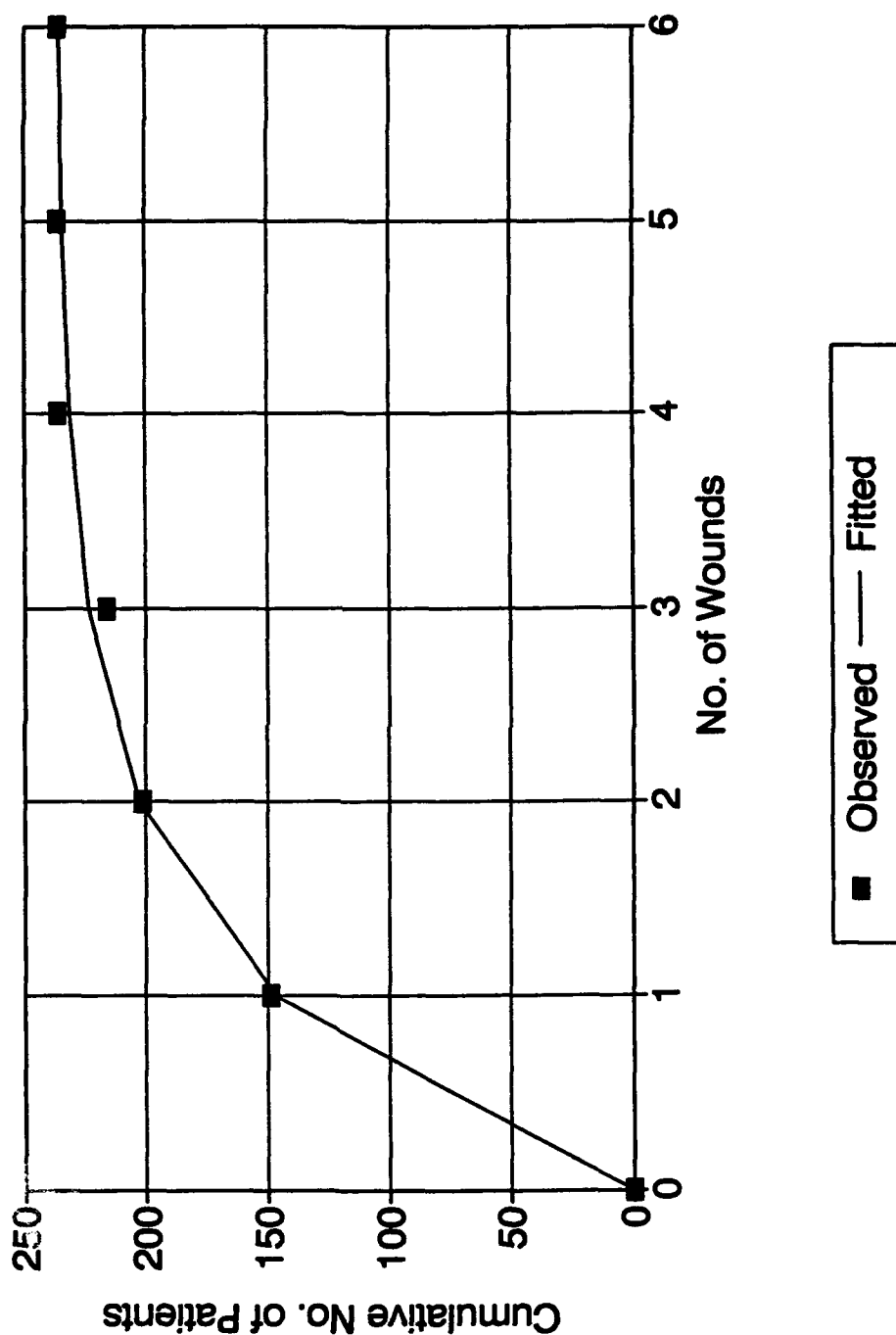


Figure 3-11. Observed and Fitted Values for McBride's Data.

Uhorchak Figure 4

Distribution of Patients by Wounds

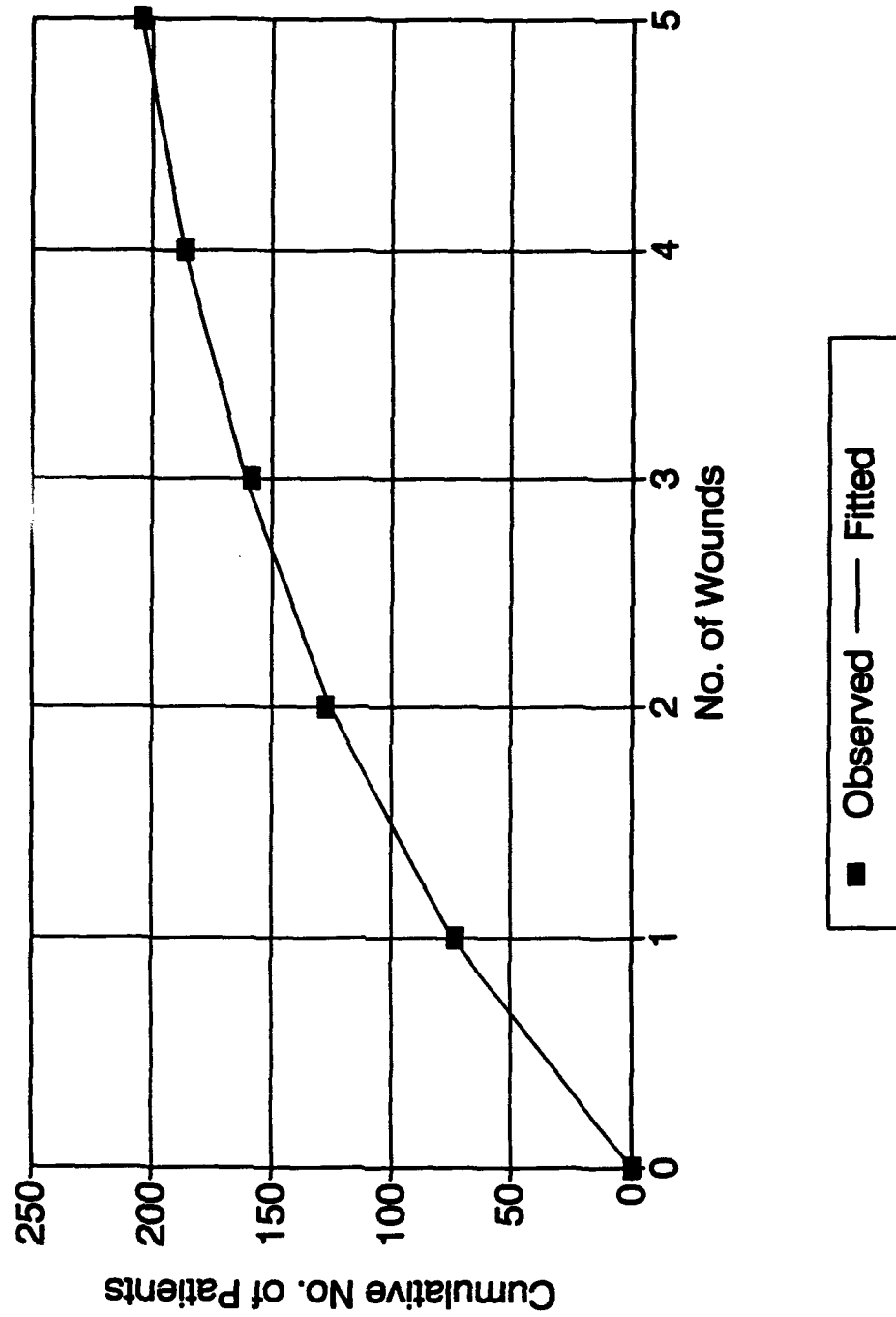


Figure 3-12. Observed and Fitted Values for Uhorchak's Data

3-5. DISTRIBUTION OF THE NUMBER OF ADMISSIONS. These examples compare the estimated number of personnel with n wounding episodes to the observed number of personnel with n wounding episodes. For these examples, the estimated number of personnel having n wounding episodes is estimated by the Maximum Entropy Principle using only the total number of personnel, the average number of wounding episodes per individual, and an assumed maximum number of wounding episodes for any particular individual.

a. The *Report of the Secretary of War to the President, 1926*, [Anonymous-1926, Table M, pp 238-239] provides information on the number of times personnel of the American Expeditionary Force were wounded during World War I. From these data, we find that 206,545 WIA individuals accounted for a total of 218,006 wounding episodes, and that no more than 4 wounding episodes were recorded for any one individual. Although the source reports that these figures include 12,934 mortal wounds, it gives no information as to how many of the mortal wounds were received on the first, second, third, or fourth wounding episode. Figure 3-13 shows the result of applying the Maximum Entropy Principle, assuming that no individual will have more than four wounding episodes. The agreement appears satisfactory.

b. Krivosheyev [Krivosheyev-1993, Table 61, p 135] gives information on the number of times personnel of the Soviet Army were wounded during World War II. The data are given separately for officers, sergeants, soldiers, and aggregate totals. In each case, these data are terminated by entries identified as "seven or more wounds." The table indicates that

- 477,077 officers accounted for a total of 862,259 wounding episodes,
- 786,526 sergeants accounted for a total of 1,404,766 wounding episodes,
- 1,405,274 soldiers accounted for a total of 2,246,490 wounding episodes,
- 2,668,877 personnel of all ranks accounted for a total of 4,513,515 wounding episodes.

Figures 3-14 through 3-17 show the results of applying the Maximum Entropy Principle, assuming that an individual may have an unlimited number of wounding episodes. The degree of agreement is satisfactory.

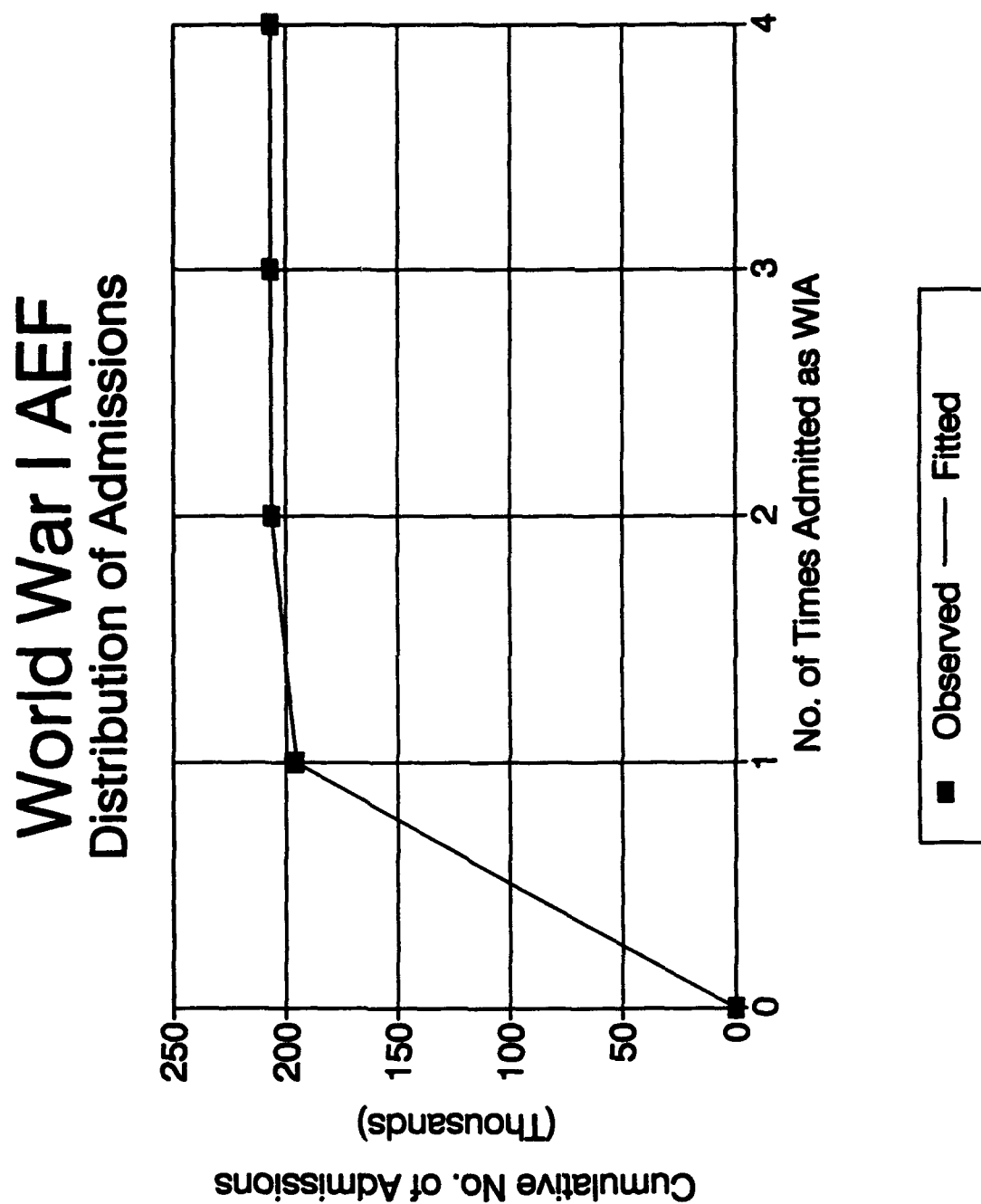


Figure 3-13. Observed and Fitted Values for AEF WIA Admissions

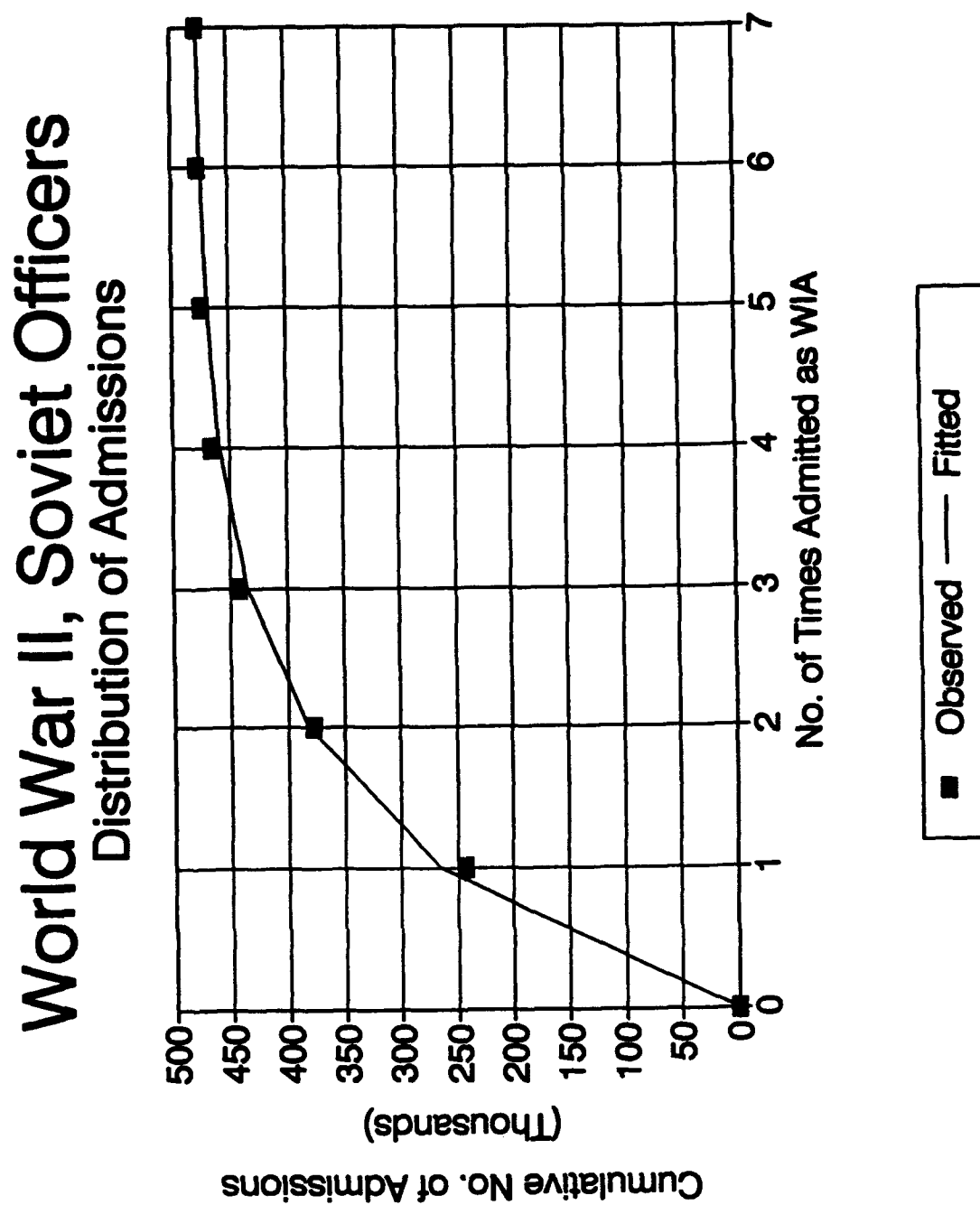


Figure 3-14. Observed and Fitted Values for Krivosheyev's Officer Data

World War II, Soviet Sergeants

Distribution of Admissions

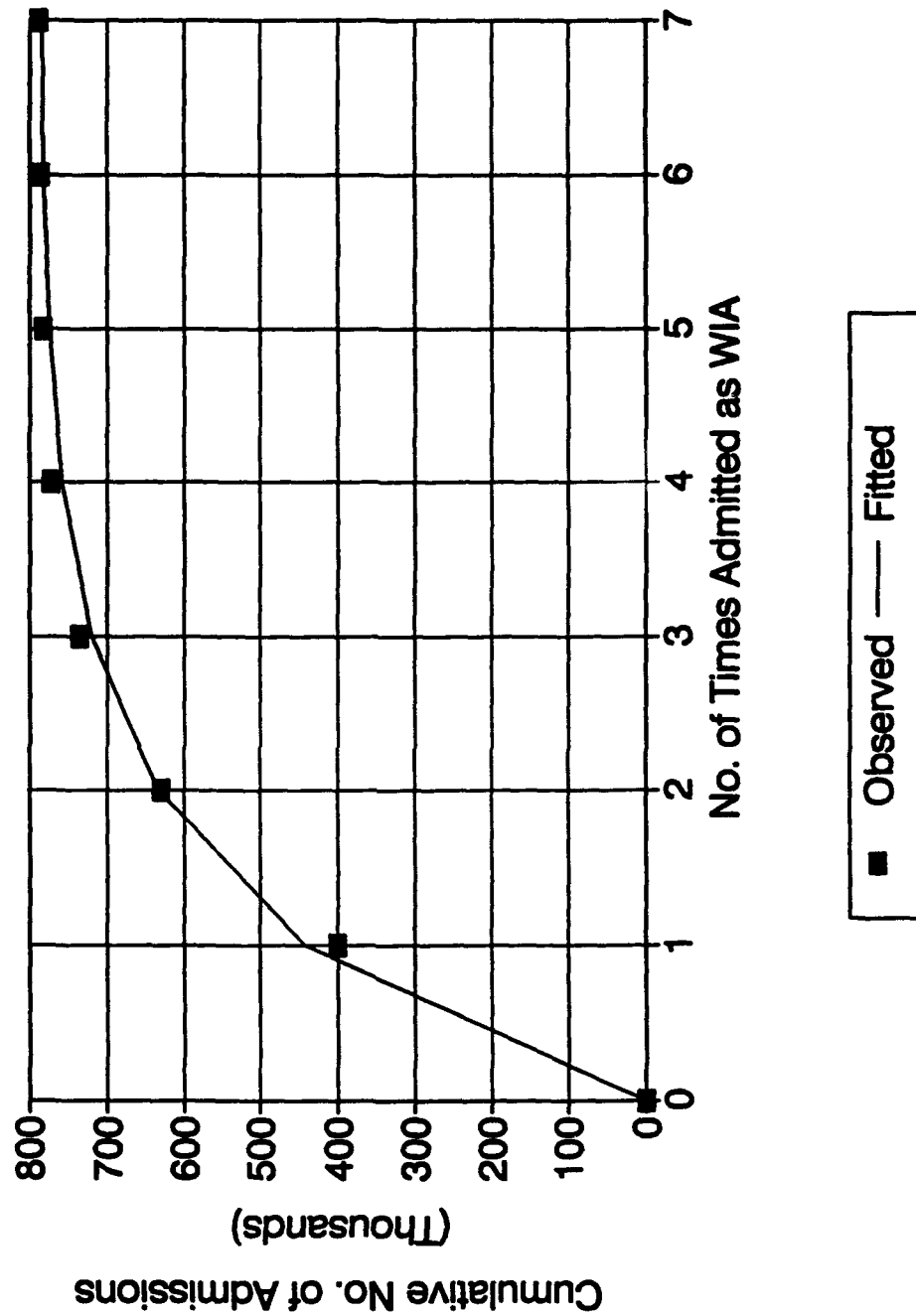


Figure 3-15. Observed and Fitted Values for Krivosheyev's Sergeant Data

World War II, Soviet Soldiers Distribution of Admissions

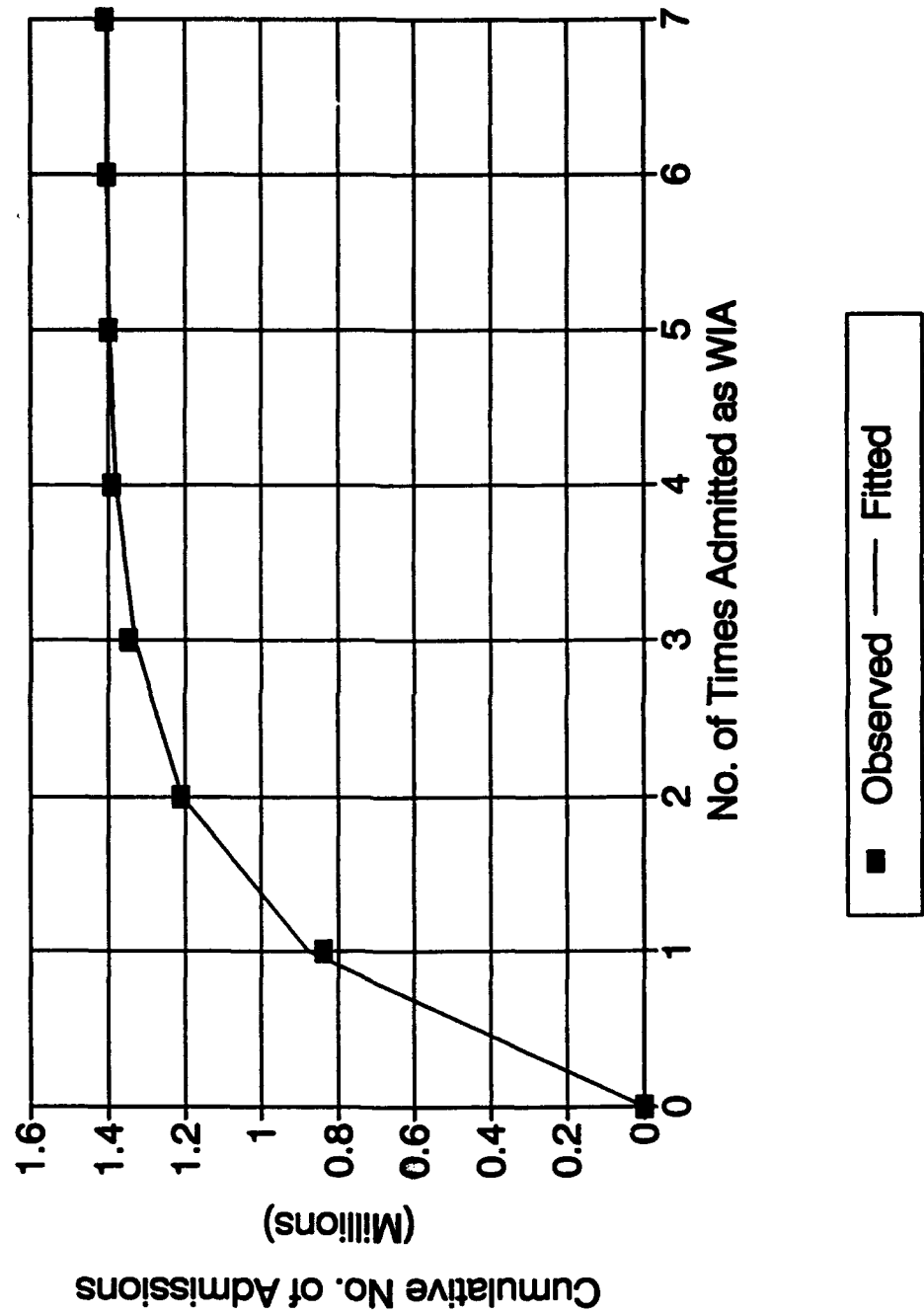


Figure 3-16. Observed and Fitted Values for Krivosheyev's Soldier Data

World War II, Soviet Aggregate Distribution of Admissions

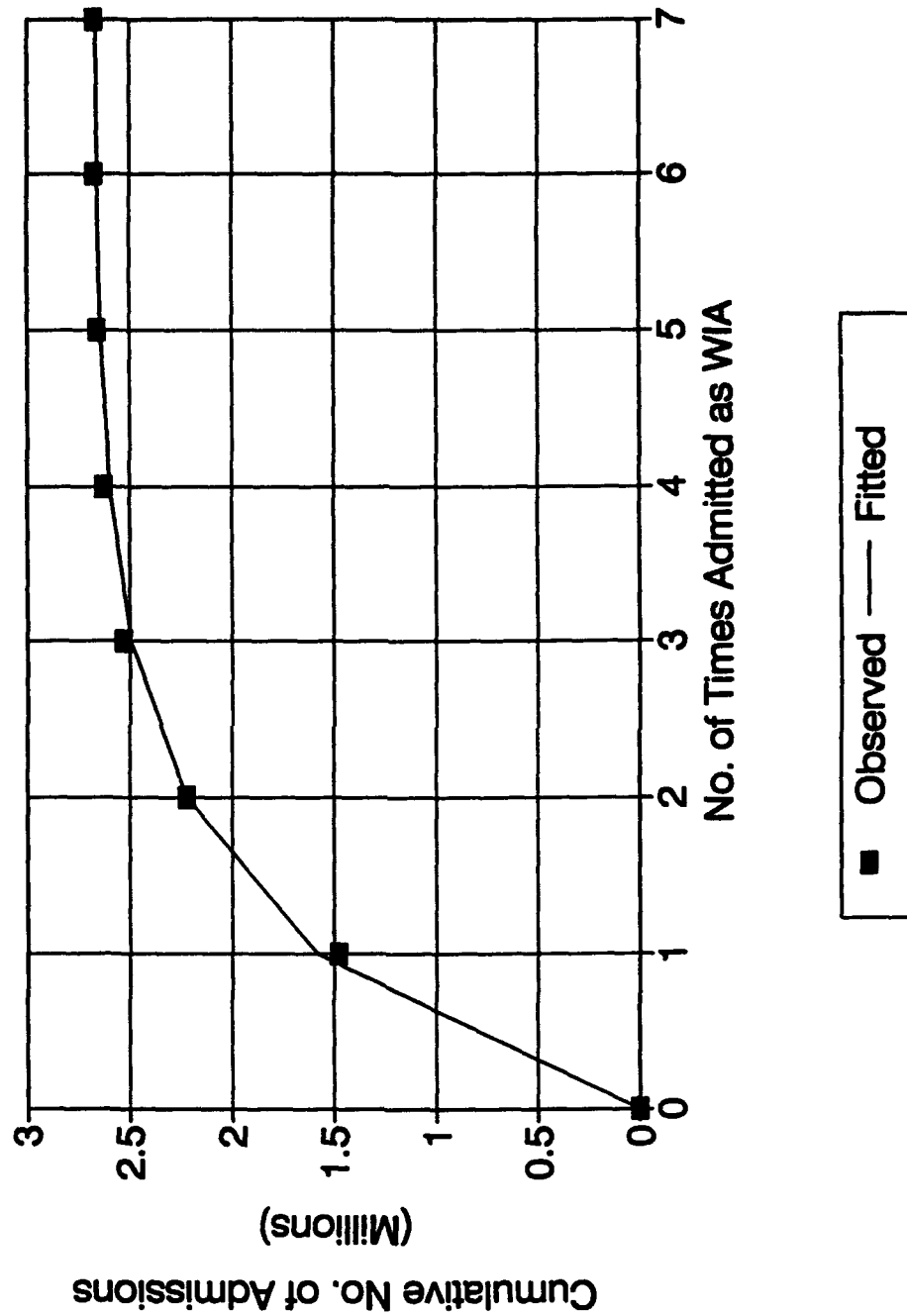


Figure 3-17. Observed and Fitted Values for Krivosheyev's Data on All Ranks

(THIS PAGE INTENTIONALLY LEFT BLANK)

CHAPTER 4

FINDINGS AND OBSERVATIONS

FINDINGS AND OBSERVATIONS. It is feasible to use published data to test the estimated distributions of multiple events obtained from the Maximum Entropy Principle. Moreover, such comparisons show that the estimated distributions obtained from the Maximum Entropy Principle often agree moderately well with historical data when such data is available for comparison. The fits occasionally would be rejected by the rote application of statistical hypothesis tests, but on the other hand these data often do not satisfy all of the assumptions required for the applicability of formal statistical hypothesis tests. At the same time, visual inspection shows that the fit is usually about as good as might be expected, and is often adequate for practical use when moderately accurate estimates are better than having no estimates. Although the results are very supportive of the applicability of the Maximum Entropy Principle to these kinds of problems, the available historical data are meager. Additional relevant data would be most welcome.

(THIS PAGE INTENTIONALLY LEFT BLANK)

APPENDIX A

BIBLIOGRAPHY

Anonymous, "Report of the Secretary of War to the President, 1926 [for the Fiscal Year ended June 30, 1926]," Prepared by the Office, US Secretary of War, 1926, 272 pp [of which only pages 191 to 240 are related to battle casualties], UNCLASSIFIED, available from the Pentagon Library (UA24.A1 1925/27).

Beyer, James C. (ed), 1962, "Wound Ballistics [Wound Ballistics in World War II, Supplemented by Experiences in the Korean War]," Medical Department, United States Army, Library of Congress catalog card number 62-60002, 1962, 883 pp. UNCLASSIFIED. Available from libraries.

CAA, 1993, "Personnel Attrition Rates in Land Combat Operations: An Annotated Bibliography," US Army Concepts Analysis Agency Research Paper, CAA-RP-93-2, June 1993.

Christenson, Ronald, *Entropy Minimax Source Book*, in 7 volumes, Entropy Limited, Lincoln, Massachusetts, 1981.

Grandy, W. T. and Milonni, P. W., *Physics and Probability: Essays in Honor of Edwin T. Jaynes*, Cambridge University Press, Cambridge, England, 1993.

Jaynes, Edwin T., "Probability Theory in Science and Engineering," Colloquium Lectures in Pure and Applied Science, No. 4, Field Research Laboratory, Socony Mobil Oil Company, Inc., February 1958.

Jaynes, Edwin T. *Papers on Probability, Statistics, and Statistical Physics*, D. Reidel, Dordrecht, Holland, 1983.

Krivosheyev, Colonel-General G. F. (ed), *Losses to the Armed Forces of the USSR in Engagements, Battles, Operations, and Military Conflicts: Statistical Analysis*, in Russian, Moscow Military Press, ISBN 5-203-01400-0, 1993, 415 pages, UNCLASSIFIED (formerly SECRET).

McBride, John T.; Hunt, Marjorie M.; Hannon, John P.; Hoxie, Stephen P.; and Rodkey, W. G., "Report and Medical Analyses of Personnel Injury from Operation 'JUST CAUSE'," Letterman Army Institute of Research, Presidio of San Francisco, Division of Military Trauma Research, Institute Report No. 468, December 1991, 89 pp. UNCLASSIFIED. Available from publisher.

Smith, Ray C., and Erickson, Gary J. (eds), *Maximum-Entropy and Bayesian Spectral Analysis and Estimation Problems*, D. Reidel, Dordrecht, Holland, 1987.

Tribus, Myron, *Thermostatistics and Thermodynamics: An Introduction to Energy, Information and States of Matter, With Engineering Applications*, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1961.

Uhorchak, John M., et al., "Final Report: Casualty Data Assessment Team Operation DESERT STORM," Letterman Army Institute of Research, Division of Military Trauma Research, Presidio of San Francisco, California 94129, Institute Report No. 469, January 1992, 115 pp. UNCLASSIFIED. Available from publisher and DTIC.

(THIS PAGE INTENTIONALLY LEFT BLANK)

APPENDIX B

REMARKS ON THE MAXIMUM ENTROPY PRINCIPLE

B-1. INTRODUCTION. This appendix provides, for the convenience of the reader, those concepts and mathematical considerations of the Maximum Entropy Principle that play a significant role in this paper. There is now a large body of material dealing with the Maximum Entropy Principle and its applications, only a small part of which is relevant to the issues involved in this paper. The interested reader can consult the following Bibliography entries for additional information regarding the Maximum Entropy Principle: Christenson-1981, Grandy-1993, Jaynes-1958, Jaynes-1983, Tribus-1961.

The following quotation from the review of Smith-1987, published in *Technometrics*, vol 31, no 2, May 1989, pp 271-272, provides an excellent summary of the Maximum Entropy Principle: "A simplified version of the problems for which maximum entropy techniques are suggested is as follows. Suppose we must assign a density function $p(\cdot)$, discrete or continuous, to a random variable X . The assignment is to be made based on some partial information concerning p . This information can take many forms—namely, either (Bayesian-like) prior information or experimental information or both. The key is that the available information does not identify p but rather specifies a class of distributions. ... If we are to still choose a single p , some criterion beyond the available, problem-specific information must be employed. The Maximum Entropy Principle states that the distribution that maximizes the expectation of $\log\{p(X)\}$ over all p satisfying the constraints dictated by the partial information should be chosen. ... The principle is defended by its proponents on the following grounds: (a) It leads to the 'least informative' distribution; (b) it is based on the laws of thermodynamics; (c) it works. The first two arguments seem to me to be questionable. ... The fact that the Maximum Entropy Principle works, although success is in the eye of the beholder, is appealing to my principles, however." In what follows, we will apply the Maximum Entropy Principle to suggest a density function for the number of admissions. (Note that maximizing the entropy is exactly equivalent to minimizing Shannon's measure of the information in the density function $p(X)$, subject to the applicable constraints. In this sense the Maximum Entropy Principle produces the least biased or informative density function consistent with the constraints.)

B-2. ILLUSTRATIVE APPLICATION TO READMISSIONS. Here we illustrate the application of the Maximum Entropy Principle to a simple model of readmissions. It is clear that the same mathematical steps apply to other cases of multiple events, such as multiple wounds. The only adjustments needed are to interpret "admissions" and "patients" suitably (for example, as "wounds" and "patients" when discussing multiple wounds). We begin by supposing that a certain group of P patients have generated a total of A hospital admissions. Here, a "patient" is an individual that has been admitted to a hospital at

least once, and only admissions for a "new" complaint are counted—i.e., readmissions for relapse from or continuing treatment of a prior admission are not counted. In this paragraph, we impose no upper limit on the number of times a patient may be admitted (see paragraph B-3 for the case of limited admissions).

a. Since each patient has been admitted at least once, $A \geq P$. If $A = P$, then we have the trivial case in which each patient has obviously been admitted exactly once. Hence, we consider only the non-trivial case, for which $A > P$. Then there are in general many ways of allocating the A admissions to the P patients. For such cases we pose the following problem: what is the expected number of patients that have been admitted exactly once? Exactly twice? Exactly three times? And so on.

b. To solve this problem, we introduce the following notation. For $k = 1$ to infinity, let f_k be the probability that a patient has been admitted exactly k times. Being probabilities, the f_k must satisfy

$$0 \leq f_k \leq 1, \text{ for all } k = 1 \text{ to infinity,} \quad (\text{B-1})$$

and must sum to unity, so that

$$\sum f_k = 1. \quad (\text{B-2})$$

where sums will be taken over all $k = 1$ to infinity, unless specifically stated otherwise.

c. Now Pf_k is the expected number of patients, each of whom has been admitted exactly k times, and so each of them contributes k admissions to the total. Accordingly, we write:

$$\sum Pkf_k = A. \quad (\text{B-3})$$

No other restrictions on the f_k are given or deducible from the problem statement. In general, the problem of finding Pf_k , the expected number of patients with exactly k admissions (or, what amounts to the same thing, the probability f_k that a patient is admitted exactly k times) is not uniquely defined by the problem statement. That is, several f_k values will satisfy the conditions prescribed by equations (B-1) through (B-3).

d. To resolve the ambiguity, we appeal to the Maximum Entropy Principle, which states that the probabilities should be selected to maximize the entropy, subject to the constraints imposed by the conditions of the problem. Here the entropy is

$$E = \sum f_k \ln f_k, \quad (\text{B-4})$$

and we proceed to maximize it subject to the conditions (B-2) and (B-3) by first forming the Lagrangian:

$$L = \sum f_k \ln f_k + \lambda(\sum kf_k - A/P) + \mu(\sum f_k - 1). \quad (\text{B-5})$$

Setting the derivative of L with respect to f_k equal to zero and solving for f_k yields

$$f_k = e^{-\nu - k\lambda}, \quad (\text{B-6})$$

where for brevity we have put $\nu = \mu + 1$. By (B-2), we have

$$1 = \sum f_k = e^{-\nu} e^{-\lambda} \sum_{k=0}^{\infty} e^{-k\lambda} = \frac{e^{-\nu} e^{-\lambda}}{1 - e^{-\lambda}} = \frac{e^{-\nu}}{e^{\lambda} - 1},$$

so that

$$e^{-\nu} = e^{\lambda} - 1, \quad (\text{B-7})$$

and hence by (B-6) and (B-7)

$$f_k = (e^{\lambda} - 1) e^{-k\lambda}. \quad (\text{B-8})$$

By (B-3) and (B-8),

$$A/P = \sum k f_k = (e^{\lambda} - 1) \sum k e^{-k\lambda}. \quad (\text{B-9})$$

To find $\sum k \exp(-k\lambda)$, consider

$$g(\lambda) = \sum e^{-k\lambda} = e^{-\lambda} \sum_{k=0}^{\infty} e^{-k\lambda} = \frac{e^{-\lambda}}{1 - e^{-\lambda}} = \frac{1}{e^{\lambda} - 1}$$

Hence

$$-g'(\lambda) = \sum k e^{-k\lambda} = \frac{e^{\lambda}}{(e^{\lambda} - 1)^2},$$

and hence

$$A/P = (e^{\lambda} - 1) \frac{e^{\lambda}}{(e^{\lambda} - 1)^2} = \frac{e^{\lambda}}{e^{\lambda} - 1}. \quad (\text{B-10})$$

Solving the last equation for e^{λ} yields

$$e^{\lambda} = \frac{A}{A - P}. \quad (\text{B-11})$$

Combining (B-11) and (B-8) yields

$$f_k = (e^{\lambda} - 1) e^{-k\lambda} = \left(\frac{A}{A - P} - 1 \right) \left(\frac{A - P}{A} \right)^k = \frac{P}{A} \left(1 - \frac{P}{A} \right)^{k-1}, \quad (\text{B-12})$$

so that k has a geometric distribution with parameter $p = P/A$, from which it is evident that the f_k satisfy the constraints (B-1).

e. By (B-6), the negentropy or information value will be

$$I = - \sum f_k \ln f_k = \sum (\nu + k\lambda) f_k = \nu \sum f_k + \lambda \sum k f_k,$$

and by (B-2) and (B-3), this is equal to

$$I = \nu + \lambda (A/P). \quad (\text{B-13})$$

Substituting for ν and λ from (B-7) and (B-11) and simplifying yields the equivalent expression

$$I = \frac{A}{P} \ln\left(\frac{A}{A-P}\right) - \ln\left(\frac{P}{A-P}\right).$$

It can be shown (by considering its derivative) that $I = I(\lambda)$, as given by (B-13), decreases with respect to λ for $0 \leq \lambda \leq \infty$. It can also be shown (by considering appropriate limits) that $I(0) = \infty$ and $I(\infty) = 0$. But, by (B-11), $\lambda > 0$, for all practical applications and hence $I(\lambda)$ must be positive for all such applications.

f. The model developed above was phrased in terminology for readmissions. However, it is apparent that the same mathematical development would apply if "admissions" were read as "wounds" and "patients" were read as "wounded in action cases." Thus, we see that in this case the Maximum Entropy Principle leads to a geometric distribution for the number of wounds suffered in a given wounded in action incident.

B-3. DEVELOPMENT WHEN THE NUMBER OF ADMISSIONS IS LIMITED. The case where the number of readmissions is limited, so that no patient will be admitted more than a certain number of times, may sometimes be more realistic in practice. For example, there may be a (formal or informal) policy that any soldier who has been wounded a certain number of times (say, three times) will be assigned to a noncombat unit even if he is fit for combat duty. In such cases, three can be taken as the maximum number of times any patient will be admitted. It will be seen that a closed solution for this case is not possible, but an iterative procedure can usually be used.

a. We begin the development by letting K be the maximum number of times a patient will be admitted. For $k = 1$ to K , let f_k be the probability that a patient has been admitted exactly k times. Being probabilities, the f_k must satisfy

$$0 \leq f_k \leq 1, \text{ for all } k = 1 \text{ to } K, \quad (\text{B-14})$$

and must sum to unity, so that

$$\sum_{k=1}^K f_k = 1. \quad (\text{B-15})$$

b. As before, $P f_k$ is the expected number of patients, each of whom has been admitted exactly k times, and so each of them contributes k admissions to the total. Accordingly, we write:

$$\sum_{k=1}^K P k f_k = A \quad (\text{B-16})$$

Here the entropy to be maximized is

$$E = \sum_{k=1}^K f_k \ln f_k, \quad (\text{B-17})$$

and we proceed to maximize it subject to the conditions (B-15) and (B-16) by forming the Lagrangian:

$$L = \sum_{k=1}^K f_k \ln f_k + \lambda \left(\sum_{k=1}^K k f_k - A/P \right) + \mu \left(\sum_{k=1}^K f_k - 1 \right). \quad (\text{B-18})$$

Setting the derivative of L with respect to f_k equal to zero and solving for f_k yields

$$f_k = e^{-\nu - k\lambda}, \quad (\text{B-19})$$

where for brevity we have put $\nu = \mu + 1$. By (B-15), we have

$$1 = \sum_{k=1}^K f_k = e^{-\nu} e^{-\lambda} \sum_{k=0}^{K-1} e^{-k\lambda} = e^{-\nu} e^{-\lambda} \frac{1 - e^{-K\lambda}}{1 - e^{-\lambda}} = e^{-\nu} \left(\frac{1 - e^{-K\lambda}}{e^{\lambda} - 1} \right),$$

so that

$$e^{-\nu} = \frac{e^{\lambda} - 1}{1 - e^{-K\lambda}}. \quad (\text{B-20})$$

Note that as $K \rightarrow \infty$, this reduces to equation (B-7). Substituting from (B-20) into (B-19) yields

$$f_k = \frac{e^{\lambda} - 1}{1 - e^{-K\lambda}} e^{-k\lambda}. \quad (\text{B-21})$$

By (B-16) and (B-21),

$$A/P = \sum_{k=1}^K k f_k = \frac{e^{\lambda} - 1}{1 - e^{-K\lambda}} \sum_{k=1}^K k e^{-k\lambda}. \quad (\text{B-22})$$

To find $\sum k \exp(-k\lambda)$, consider

$$g(\lambda) = \sum_{k=1}^K e^{-k\lambda} = e^{-\lambda} \sum_{k=0}^{K-1} e^{-k\lambda} = \frac{1 - e^{-K\lambda}}{e^{\lambda} - 1},$$

Hence

$$-g'(\lambda) = \sum_{k=1}^K k e^{-k\lambda} = \frac{1 - e^{-K\lambda}}{(e^{\lambda} - 1)^2} e^{\lambda} - \frac{K e^{-K\lambda}}{e^{\lambda} - 1}, \quad (\text{B-23})$$

and hence, after some easy simplification,

$$A/P = \frac{e^{\lambda}}{e^{\lambda} - 1} - \frac{K}{e^{K\lambda} - 1}. \quad (\text{B-24})$$

c. We observe two things about equation (B-24). First, as $K \rightarrow \infty$ it reduces to equation (B-10). Second, unlike equation (B-10), it has no closed solution for λ in terms of K and A/P . Consequently, either an iterative process must be used to find the value of λ , or else λ must be varied parametrically. In the latter approach, λ is viewed as determining the value of A/P , rather than the other way around, and this can sometimes be satisfactory in practice. If an iterative scheme for approximating the value of λ corresponding to a given value of A/P is desired, then we suggest the following approach. Begin by rewriting equation (B-24) as

$$\frac{e^\lambda}{e^\lambda - 1} = A/P + \frac{K}{e^{K\lambda} - 1},$$

and solve this for e^λ in terms of the right-hand side to obtain

$$e^\lambda = \frac{A + C_K(e^\lambda)}{A - P + C_K(e^\lambda)}, \quad (\text{B-25})$$

where we have set

$$C_K(e^\lambda) = \frac{PK}{e^{K\lambda} - 1}.$$

Note that, as $K \rightarrow \infty$ for a fixed λ and P , $C_K(e^\lambda)$ approaches zero, and hence (B-25) approaches (B-11). Now, (B-25) is suitable for iterative computations, and the following procedure will often suffice. First, decide on some value of ϵ , a small numerical tolerance value. We will also use x as an abbreviation for e^λ .

(i) Begin the iterative process by setting $x = \frac{A}{A-P}$, the value that would obtain if K were infinite.

(ii) Find a new value, x' , by setting

$$C_K(x) = \frac{PK}{x^K - 1}, \text{ and then putting}$$

$$x' = \frac{A + C_K(x)}{A - P + C_K(x)}.$$

(iii) If the absolute fractional difference between the new and old values exceeds the tolerance value, $\left| \frac{x - x'}{x'} \right| > \epsilon$, then set $x = x'$, and return to step (ii).

(iv) Otherwise, terminate the iteration and take $\lambda = \ln(x')$.

d. In any case, by using (B-19), the negentropy or information value will be given by

$$I = - \sum_{k=1}^K f_k \ln f_k = \sum_{k=1}^K (\nu + k\lambda) f_k = \nu \sum_{k=1}^K f_k + \lambda \sum_{k=1}^K k f_k,$$

which by (B-15) and (B-16) is equal to

$$I = \nu + \lambda A/P.$$

where ν is given by equation (B-20) and λ by equation (B-24).

e. Although this model is phrased in terminology for readmissions, it is apparent that the same mathematical development would apply if "admissions" were read as "wounds" and "patients" were read as "wounded in action cases." Thus, we see that in this case the Maximum Entropy Principle leads to a distribution for the number of wounds suffered in a given wounded in action incident.

B-4. REMARKS ON SOME SPECIAL CASES. When the number of admissions per patient is limited, it is not possible to prescribe an arbitrary number of admissions and patients. For example, it is not possible to limit the number of admissions per patient to $K = 2$, and at the same time to impose the constraint that the admission/patient ratio $A/P = 3$, for these requirements are mutually contradictory. In fact, it is generally true that we must have $A/P \leq K$. When K is infinite, this constraint is not binding, but when K is finite, it may be.

In fact, when A/P is close to K , the Lagrange multiplier method may fail. For example, when $A/P = K$, there is only one possible solution, which is entirely defined by the constraints imposed and is independent of any optimization criteria. The reason for this is as follows. By the constraints, we have $0 \leq f_k \leq 1$, $\sum f_k = 1$, and $\sum k f_k = A/P = K$. Under these conditions, $\sum k f_k$ is a convex linear combination of the integers k from 1 to K , and so cannot possibly exceed the largest of those integers. Consequently, $\sum k f_k \leq K$. It is easy to see that the only possible density function for which $\sum k f_k = K$ is the one for which $f_K = 1$ and all the other f_k vanish. However, this is not a density function of the form $f_k = e^{-\nu - \lambda k}$, as required by the Lagrange multiplier method, and so the Lagrange multiplier method fails in this case.

Similarly, it can be shown that when $K = 3$ and $A/P = 2$, the only feasible solution for the density function is $f_1 = f_2 = f_3 = 1/3$, regardless of any optimization considerations. This example illustrates the more general finding of the numerical experiments we have made, which is that either the Lagrange multiplier method or our proposed iterative scheme may fail to produce suitable solutions whenever K is less than twice the admission/patient ratio, A/P . We have not inquired closely into the reasons for this, as that would take us too far afield, but it is a caution that potential users of this method should bear in mind.

(THIS PAGE INTENTIONALLY LEFT BLANK)

APPENDIX C DISTRIBUTION

Addressee	No. of copies
Deputy Under Secretary of the Army (Operations Research), Room 2E660 The Pentagon Washington, DC 20310	1
Deputy Chief of Staff for Operations and Plans Headquarters, Department of the Army ATTN: DAMO-ZXA Washington, DC 20310	1
Deputy Chief of Staff for Logistics Headquarters, Department of the Army ATTN: DALO-ZXA-A Room 3D572, The Pentagon Washington, DC 20310-0580	1
Commander Combined Arms Support Command ATTN: ATCL-CFC Fort Lee, VA 23801-6000	1
Office of the Secretary of the Army Correspondence & Records Center Management Systems & Support ATTN: JDMSS-CRC ROOM 3D718, The Pentagon Washington, DC 20310-0105	1
Office of The Surgeon General ATTN: DASG-HCD 5109 Leesburg Pike Falls Church, VA 22041-3258	1
Director US Army TRADOC Analysis Command-WSMR ATTN: ATRC-WSL White Sands Missile Range, NM 88002-5502	1
Commander, TRAC ATTN: ATRC-TD Fort Leavenworth, KS 66027-5200	1
HQ TRADOC ATTN: ATAN-S Fort Monroe, VA 22651-5143	1

Addressee	No. of copies
Director US Army Materiel Systems Analysis Activity ATTN: AMXSY-LM Aberdeen Proving Ground, MD 21005-5071	1
Director US Army Research Laboratories ATTN: AMSRL-SL-B (Klopacic) Aberdeen Proving Ground, MD 21005-5068	1
Commander US Total Army Personnel Agency ATTN: DAPC-ZA 200 Stovall Street Alexandria, VA 22332	1
Defense Technical Information Center ATTN: DTIC-FDAC Cameron Station Alexandria, VA 22314-6145	2
USASCAF The Pentagon Library ATTN: JDHQ-LR (Army Studies) Room 1A518, The Pentagon Washington, DC 20310-6000	1
Commandant US Army War College Operations Group ATTN: AWCMA Carlisle Barracks, PA 17013-5050	1
Air War College ATTN: AU/CADRE/WGOI Maxwell Air Force Base, AL 36112-5532	1
President US Navy War College ATTN: Code E-111/Library Newport, RI 02841-5010	1
President National Defense University ATTN: NDU-LD-CDC Washington, DC 20319-6000	1
Commandant Armed Forces Staff College ATTN: Library Room B-201 7800 Hampton Blvd Norfolk, VA 23511-3097	1

Addressee	No. of copies
Commandant US Army Command and General Staff College ATTN: ATZL-SWS-L (Mail) Fort Leavenworth, KS 66027-6900	1
Superintendent Naval Postgraduate School ATTN: Security Manager Monterey, CA 93940	1
Commander US Army Health Services Command ATTN: HSOP-FSI Fort Sam Houston, TX 78234-6000	1
Commander US Army Medical Research and Development Command ATTN: SGRD-OP (Mr. Adams) Fort Detrick, MD 21701	1
Director Strategic Studies Institute ATTN: AWCI Carlisle Barracks, PA 17013-5050	1
Commander/Director US Army Engineer Studies Center Casey Building, No. 2594 ATTN: ESC-AO (Security Officer) Fort Belvoir, VA 22060-5583	1
Commander US Army Training and Doctrine Command ATTN: ATIM-OPM Fort Monroe, VA 23651-5000	1
Headquarters Tactical Air Command ATTN: DAAS Langley Air Force Base, VA 23665-5001	1
Commandant Air Force Institute of Technology ATTN: AFIT-EN Wright-Patterson AFB, OH 45433	1
Chief of Naval Operations ATTN: OP-09B3471 Room 4C479, The Pentagon Washington, DC 20350	1

Addressee	No. of copies
United States General Accounting Office (GAO) National Security and International Affairs Division Washington, DC 20548	1
Director Program Analysis and Evaluation (PA&E) Office of the Secretary of Defense Room 2E330 The Pentagon, Washington, DC 20310	1
The Borden Institute Textbook of Military Medicine ATTN: COL Ronald F. Bellamy Walter Reed Army Medical Center Building 1, Room D-100 Washington DC 20307-5001	1
Internal Distribution:	
Reference copy: Unclassified Library	2
Record copy: Originating office (CSCA-TCT)	1
Other copies: Dr. Robert L. Helmbold	20

GLOSSARY

GLOSSARY-1. Some of the abbreviations and special terms used in this document are listed below. If the definition given is a official one, the organizations that have adopted it are given in parentheses; otherwise, no indication of its adoption are given. Note that the definitions used by other countries or by the US in earlier times may differ more or less from those given below, and may be interpreted in various ways even within the US Department of Defense.

GLOSSARY-2. Definitions of terms and abbreviations.

Battle casualty.- (DOD) Any casualty incurred in action. "In action" characterizes the casualty status as having been the direct result of hostile action, sustained in combat or relating thereto, or sustained going to or returning from a combat mission provided that the occurrence was directly related to hostile action. Included are persons killed or wounded mistakenly or accidentally by friendly fire directed at a hostile force or what is thought to be a hostile force. However, not to be considered as sustained in action and thereby not to be interpreted as battle casualties are injuries due to the elements, self-inflicted wounds, and, except in unusual cases, wounds or death inflicted by friendly forces while the individual is in absent-without-leave or dropped-from-rolls status or is voluntarily absent from a place of duty. See also died of wounds received in action; nonbattle casualty; wounded.

Bloody losses.- The sum of the KIA and WIA.

Casualty.- (DOD, IADB) Any person who is lost to the organization by reason of having been declared dead, wounded, injured, diseased, interned, captured, retained, missing, missing in action, beleaguered, besieged or detained; see also battle casualty; nonbattle casualty; wounded.

CMIA.- Captured or missing in action. See POW and MIA.

CRO.- Carded for record only. (Adapted from Beebe, Gilbert W.; and De Bakey, Michael E., *Battle Casualties: Incidence, Mortality, and Logistic Considerations*. Charles C. Thomas (publisher), 1952.) Basically, admissions to a medical treatment facility include all cases admitted for medical care and not returned to duty on the same calendar day as that on which first seen. Cases which are treated on an outpatient (duty) status, are designated as carded for record only (CRO).

DNBI.- Disease and nonbattle injury. Personnel treated for diseases and for injuries not received in action. See Nonbattle casualty.

DOW.- Died of wounds received in action (DOD, NATO). A battle casualty who dies of wounds or other injuries received in action, after having reached a medical treatment facility. See also killed in action.

DTIC.- Defense Technical Information Center.

KIA.- Killed in action (DOD, NATO, IADB). A battle casualty who is killed outright or who dies as a result of wounds or other injuries before reaching a medical treatment facility. See also died of wounds received in action.

Losses.- (Adapted from FM 101-10-1/2, Staff Officers' Field Manual Organizational, Technical, and Logistical Data Planning Factors, October 1987.) A personnel loss is any reduction in the assigned strength of a unit. Personnel losses are recorded in three general categories: battle, nonbattle, and administrative.

- Battle losses are those incurred in action. They include wounded or injured in action (including those who died of wounds and died of injuries received in action), killed in action, and missing in action or captured by the enemy.

- Nonbattle losses are those not directly attributable to action regardless of when sustained. They include nonbattle dead, nonbattle accident/injury, nonbattle missing, and illness/disease.

- Administrative losses are those resulting from transfer from the unit, absence without leave, desertion, personnel rotation, and discharges.

LWIA.- Lightly wounded in action (*cf.* Slightly Wounded).

MIA.- (adapted from FM 101-10-1/2, Staff Officers' Field Manual Organizational, Technical, and Logistical Data Planning Factors, October 1987). Missing in action describes battle casualties whose whereabouts or fate cannot be determined and who are not known to be in an unauthorized absence status (desertion or absence without leave). Missing in action (MIA) casualties are not usually included in medical statistical records or reports received by The Surgeon General, but are reportable to The Adjutant General.

NFW.- Nonfatal wound. A person who is wounded in action (WIA), but who does not die of wounds (DOW).

Nonbattle casualty.- (DOD, NATO, IADB) A person who is not a battle casualty, but who is lost to his organization by reason of disease or injury, including persons dying from disease or injury, or by reason of being missing where the absence does not appear to be voluntary or due to enemy action. See also battle casualty; wounded.

Nonbloody loss.- Battle casualties other than KIA and WIA; includes (for example) MIA, POW, absent without leave, stragglers, and deserters.

NP.- Neuropsychiatric.

POW.- Prisoner of war. Detainee (DOD). A term used to refer to any person captured or otherwise detained by an armed force. (According to FM 101-10-1/2, Staff Officers' Field Manual Organizational, Technical, and Logistical Data Planning Factors, October 1987, captured describes all battle casualties known to have been taken into custody by a hostile force as a result of and for reasons arising out of any armed conflict in which US armed forces are engaged. Captured casualties are not usually included in medical statistical records or reports received by The Surgeon General but are reported to The Adjutant General.)

Seriously wounded.- (DOD, IADB) A stretcher case. See also WIA.

Slightly wounded.- (DOD, IADB) A casualty that is a sitting or walking case. See also WIA.

SWIA.- Seriously wounded in action (*cf.* Seriously Wounded).

WIA.- Wounded in action (DOD, NATO, IADB). A battle casualty other than "killed in action" who has incurred an injury due to an external agent or cause. The term encompasses all kinds of wounds and other injuries incurred in action, whether there is a piercing of the body, as in a penetrating or perforated wound, or none, as in the contused wound; all fractures, burns, blast concussions, all effects of biological and chemical warfare agents, the effects of exposure to ionizing radiation, or any other destructive weapon or agent.